GRADE 7 MATH LEARNING GUIDE

Lesson 26: Solving Linear Equations and Inequalities in One Variable Using Guess and Check **Time: 1 hour**

Prerequisite Concepts: Evaluation of algebraic expressions given values of the variables

About the Lesson: This lesson will deal with finding the unknown value of a variable that will make an equation true (or false). You will try to prove if the value/s from a replacement set is/are solution/s to an equation or inequality. In addition, this lesson will help you think logically via guess and check even if rules for solving equations are not yet introduced.

Objective:
In this lesson, you are expected to:
1. Differentiate between mathematical expressions and mathematical equations.
2. Differentiate between equations and inequalities.
3. Find the solution of an equation and inequality involving one variable from a given replacement set by guess and check.

Lesson Proper:
I. Activity
A mathematical expression may contain variables that can take on many values. However, when a variable is known to have a specific value, we can substitute this value in the expression. This process is called *evaluating a mathematical expression.* Instructions: Evaluate each expression under Column A if $x = 2$. Match it to its value under Column B and write the corresponding letter on the space before each item. A passage will be revealed if answered correctly.

COLUMN A		COLUMN B	
______ 1.	$3 + x$	A.	-3
______ 2.	$3x - 2$	C.	-1
______ 3.	$x - 1$	E.	-5
______ 4.	$2x - 9$	F.	1
______ 5.	$\frac{1}{2}x + 3$	H.	-2
______ 6.	$5x$	I.	4
______ 7.	$x - 5$	L.	5
______ 8.	$1 - x$	O.	6
______ 9.	$-4 + x$	S.	10
______ 10.	$3x$		
______ 11.	$14 - 5x$		
______ 12.	$-x + 1$		
______ 13.	$1 - 3x$		

PASSAGE: "____"

II. Activity

Mental Arithmetic: How many can you do orally?

1)	$2(5) + 2$	6)	$5(4)$
2)	$3(2 - 5)$	7)	$2(5 + 1)$
3)	$6(4 + 1)$	8)	$-9 + 1$
4)	$-(2 - 3)$	9)	$3 + (-1)$
5)	$3 + 2(1 + 1)$	10)	$2 - (-4)$

III. Activity

Directions: The table below shows two columns, A and B. Column A contains mathematical expressions while Column B contains mathematical equations. Observe the items under each column and compare. Answer the questions that follow.

Column A	**Column B**
Mathematical Expressions	Mathematical Equations
$x + 2$	$x + 2 = 5$
$2x - 5$	$4 = 2x - 5$
x	$x = 2$
7	$7 = 3 - x$
____________	____________
____________	____________

1) How are items in Column B different from Column A?

2) What symbol is common in all items of Column B?

3) Write your own examples (at least 2) on the blanks provided below each column.

Directions: In the table below, the first column contains a mathematical expression, and a corresponding mathematical equation is provided in the third column. Answer the questions that follow.

Mathematical Expression	**Verbal Translation**	**Mathematical Equation**	**Verbal Translation**
$2x$	five added to a number	$2x = x + 5$	Doubling a number gives the same value as adding five to the number.
$2x - 1$	twice a number decreased by 1	$1 = 2x - 1$	1 is obtained when twice a number is decreased by 1.
$7 + x$	seven increased by a number	$7 + x = 2x + 3$	Seven increased by a number is equal to twice the same number increased by 3.
$3x$	thrice a number	$3x = 15$	Thrice a number x gives 15.
$x - 2$	two less than a number	$x - 2 = 3$	Two less than a number x results to 3.

1) What is the difference between the verbal translation of a mathematical expression from that of a mathematical equation?

2) What verbal translations for the "=" sign do you see in the table? What other words can you use?

3) Can we evaluate the first mathematical expression ($x + 5$) in the table when $x = 3$? What happens if we substitute $x = 3$ in the corresponding mathematical equation ($x + 5 = 2x$)?

4) Can a mathematical equation be true or false? What about a mathematical expression?

5) Write your own example of a mathematical expression and equation (with verbal translations) in the last row of the table.

IV. Activity

From the previous activities, we know that a mathematical equation with one variable is similar to a complete sentence. For example, the equation $x – 3 = 11$ can be expressed as, "Three less than a number is eleven." This equation or statement may or may not be true, depending on the value of x. In our example, the statement $x – 3 = 11$ is true if $x = 14$, but not if $x = 7$. We call $x = 14$ a *solution* to the mathematical equation $x – 3 = 11$.

In this activity, we will work with mathematical inequalities which, like a mathematical equation, may either be true or false. For example, $x – 3 < 11$ is true when $x = 5$ or when $x = 0$ but not when $x = 20$ or when $x = 28$. We call all possible x values (such as 5 and 0) that make the inequality true *solutions* to the inequality.

Complete the following table by placing a check mark on the cells that correspond to x values that make the given equation or inequality true.

	$x = –4$	$x = –1$	$x = 0$	$x = 2$	$x = 3$	$x = 8$
$0 = 2x + 2$						
$3x + 1 < 0$						
$–1 \geq 2 – x$						
$\frac{1}{2}(x - 1) = –1$						

1) In the table, are there any examples of linear equations that have more than one solution?

2) Do you think that there can be more than one solution to a linear inequality in one variable? Why or why not?

V. Questions/Points to Ponder

In the previous activity, we saw that linear equations in one variable may have a unique solution, but linear inequalities in one variable may have many solutions. The following examples further illustrate this idea.

Example 1. Given, $x + 5 = 13$, prove that only one of the elements of the replacement set
{–8, –3, 5, 8, 11} satisfies the equation.

$x + 5 = 13$				
For $x = -8$: $-8 + 5 = -3$ $-3 \neq 13$ Therefore –8 is not a solution.	For $x = -3$: $-3 + 5 = 2$ $2 \neq 13$ Therefore –3 is not a solution.	For $x = 5$: $5 + 5 = 10$ $10 \neq 13$ Therefore 5 is not a solution.	For $x = 8$: $8 + 5 = 13$ $13 = 13$ Therefore 8 is a solution.	For $x = 11$: $11 + 5 = 16$ $16 \neq 13$ Therefore 11 is not a solution.

Based on the evaluation, only $x = 8$ satisfied the equation while the rest did not. Therefore, we proved that only one element in the replacement set satisfies the equation.

We can also use a similar procedure to find solutions to a mathematical inequality, as the following example shows.
Example 2. Given, $x - 3 \leq 5$, determine the element/s of the replacement set {–8,–3, 5, 8, 11} that satisfy the inequality.

$x - 3 \leq 5$				
For $x = -8$: $-8 - 3 = -11$ $-11 \leq 5$ Therefore –8 is a solution.	For $x = -3$: $-3 - 3 = -6$ $-6 \leq 5$ Therefore –3 is a solution.	For $x = 5$: $5 - 3 = 2$ $2 \leq 5$ Therefore 5 is a solution.	For $x = 8$: $8 - 3 = 5$ $5 \leq 5$ Therefore 8 is a solution.	For $x = 11$: $11 - 3 = 8$ $8 \not\leq 13$ Therefore 11 is not a solution.

Based on the evaluation, the inequality was satisfied if $x = -8, -3, 5,$ or 8. The inequality was not satisfied when $x = 11$. Therefore, there are 4 elements in the replacement set that are solutions to the inequality.

VI. Exercises

Given the replacement set {–3, –2, –1, 0, 1, 2, 3}, determine the solution/s for the following equations and inequalities. Show your step-by-step computations to prove your conclusion.

1) $x + 8 < 10$
2) $2x + 4 = 3$
3) $x - 5 > -3$
4) $x > -4$ and $x \leq 2$
5) $x < 0$ and $x > 2.5$

Solve for the value of x to make the mathematical sentence true. You may try several values for x until you reach a correct solution.

1) $x + 6 = 10$
2) $x - 4 = 11$
3) $2x = 8$
4) $\frac{1}{5}x = 3$
5) $5 - x = 3$
6) $4 + x = 9$
7) $-4x = -16$
8) $-\frac{2}{3}x = 6$
9) $2x + 3 = 13$
10) $3x - 1 = 14$

VII. Activity

Match the solutions under Column B to each equation or inequality in one variable under Column A. Remember that in inequalities there can be more than one solution.

COLUMN A		COLUMN B	
_____ 1.	$3 + x = 4$	A.	-9
_____ 2.	$3x - 2 = 4$	B.	-1
_____ 3.	$x - 1 < 10$	C.	-5
_____ 4.	$2x - 9 \geq -7$	D.	1
_____ 5.	$\frac{1}{2}x + 3 = -3$	E.	-2
_____ 6.	$2x > -10$	F.	4
_____ 7.	$x - 5 = 13$	G.	-4
_____ 8.	$1 - x = 11$	H.	6
_____ 9.	$-3 + x > 1$	I.	10
_____ 10.	$-3x = 15$	J.	2
_____ 11.	$14 - 5x \leq -1$	K.	18
_____ 12.	$-x + 1 = 10$	L.	11
_____ 13.	$1 - 3x = 13$	M.	-10
		N.	3
		O.	-12

VIII. Activity

Scavenger Hunt. You will be given only 5-10 minutes to complete this activity. Go around the room and ask your classmates to solve one task. They should write the correct answer and place their signature in a box. Each of your classmates can sign in at most two boxes. You cannot sign on own paper. Also, when signing on your classmates' papers, you cannot always sign in the same box.

Find someone who

Can give the value of x so that $x + 3 = 5$ is a true equation.	**Can determine the smallest integer value for x that can hold $x > 1.5$ true.**	**Can solve by guess and check for the solution of $9x–1=8$.**	**Can give the value of $3x–1$ if $x = 3$.**
Can give the numerical value of $3(2^2 – 3^2)$.	**Knows which is greater between x^3 and 3^x when $x = 2$.**	**Can translate the phrase 'a number x increased by 3 is 2' to algebraic expression.**	**Can determine which of these {0,1, 2,…, 8, 9} is/are solution/s of $3x < 9$.**
Can write an inequality for which all positive numbers are NOT solutions.	**Knows the largest integer value of x that can satisfy the inequality $2x – 1 < 3$?**	**Knows what Arabic word is known to be the origin of the word Algebra.**	**Can write an equation that is true when $x = 4$.**
Can write a simple inequality that will is satisfied by the elements in the set $\{–1, 0, 1.1, \sqrt{2}, 3, 4, …\}$.	**Can name the set of numbers satisfying the inequality $x < 0$.**	**Can explain what an open sentence is.**	**Can give the positive integer values of x that can satisfy $x + 3 < 6$.**

Summary

In this lesson, you learned how to evaluate linear equations at a specific value of *x*. You also learned to determine whether particular values of *x* are solutions to linear equations and inequalities in one variable.

Lesson 27: Solving Linear Equations and Inequalities Algebraically

Time: 2 hours

Prerequisite Concepts: Operations on polynomials, verifying a solution to an equation

About the Lesson: This lesson will introduce the properties of equality as a means for solving equations. Furthermore, simple word problems on numbers and age will be discussed as applications to solving equations in one variable.

Objectives:

In this lesson, you are expected to:

1. Identify and apply the properties of equality
2. Find the solution of an equation involving one variable by algebraic procedure using the properties of equality
3. Solve word problems involving equations in one variable

Lesson Proper:

I. Activity1

The following exercises serve as a review of translating between verbal and mathematical phrases, and evaluating expressions.

Instructions: Answer each part neatly and promptly.

A. Translate the following verbal sentences to mathematical equation.
   1. The difference between five and two is three.
   2. The product of twelve and a number *y* is equal to twenty-four.
   3. The quotient of a number *x* and twenty-five is one hundred.
   4. The sum of five and twice *y* is fifteen.
   5. Six more than a number *x* is 3.

B. Translate the following equations to verbal sentences using the indicated expressions.
   1. $a + 3 = 2$, “the sum of”
   2. $x - 5 = 2$, “subtracted from”
   3. $\frac{2}{3}x = 5$, “of”
   4. $3x + 2 = 8$, “the sum of”
   5. $6b = 36$, “the product of”

C. Evaluate $2x + 5$ if:
   1. $x = 5$
   2. $x = -4$
   3. $x = -7$
   4. $x = 0$
   5. $x = 13$

II. Activity

The Properties of Equality. To solve equations algebraically, we need to use the various properties of equality. **Create your own examples for each property.**

A. Reflexive Property of Equality

For each real number a, $a = a$.

Examples: $3 = 3$ $\quad$ $-b = -b$ $\quad$ $x + 2 = x + 2$

B. Symmetric Property of Equality

For any real numbers a and b, if $a = b$ then $b = a$.

Examples: If $2 + 3 = 5$, then $5 = 2 + 3$.
If $x - 5 = 2$, then $2 = x - 5$.

C. Transitive Property of Equality

For any real numbers a, b, and c,
If $a = b$ and $b = c$, then $a = c$

Examples: If $2 + 3 = 5$ and $5 = 1 + 4$, then $2 + 3 = 1 + 4$.
If $x - 1 = y$ and $y = 3$, then $x - 1 = 3$.

D. Substitution Property of Equality

For any real numbers a and b: If $a = b$, then a may be replaced by b, or b may be replaced by a, in any mathematical sentence without changing its meaning.

Examples: If $x + y = 5$ and $x = 3$, then $3 + y = 5$.
If $6 - b = 2$ and $b = 4$, then $6 - 4 = 2$.

E. Addition Property of Equality (APE)

For all real numbers a, b, and c,
$a = b$ if and only if $a + c = b + c$.

If we add the same number to both sides of the equal sign, then the two sides remain equal.

Example: $10 + 3 = 13$ is true if and only if $10 + 3 \mathbf{+ 248} = 13 \mathbf{+ 248}$ is also true (because the same number, 248, was added to both sides of the equation).

F. Multiplication Property of Equality (MPE)

For all real numbers a, b, and c, where $c \neq 0$,
$a = b$ if and only if $ac = bc$.

If we multiply the same number to both sides of the equal sign, then the two sides remain equal.

Example: $3 \cdot 5 = 15$ is true if and only if $(3 \cdot 5) \mathbf{\cdot 2} = 15 \mathbf{\cdot 2}$ is also true (because the same number, 2, was multiplied to both sides of the equation).

Why is there no Subtraction or Division Property of Equality? Even though subtracting or dividing the same number from both sides of an equation preserves equality, these cases are already covered by APE and MPE. Subtracting the same number from both sides of an equality is the same as *adding* a negative number to both sides of an equation. Also, dividing the same number from both sides of an equality is the same as *multiplying* the reciprocal of the number to both sides of an equation.

III. Exercises

Directions: Answer the following exercises neatly and promptly.

A. Identify the property shown in each sentence.
   1. If $3 \cdot 4 = 12$ and $12 = 2 \cdot 6$. then $3 \cdot 4 = 2 \cdot 6$
   2. $12 = 12$
   3. If $a + 2 = 8$, then $a + 2 + (–2) = 8 + (–2)$.
   4. If $1 + 5 = 6$, then $6 = 1 + 5$.
   5. If $3x = 10$, then $\frac{1}{3}(3x) = \frac{1}{3}(10)$

B. Fill-in the blanks with correct expressions indicated by the property of equality to be used.
   1. If $2 + 5 = 7$, then $7 =$ ____ (Symmetric Property)
   2. $(80 + 4) \cdot 2 = 84 \cdot$ ____ (Multiplication Property)
   3. $11 + 8 = 19$ and $19 = 10 + 9$, then $11 + 8 =$ _____ (Transitive Property)
   4. $3 + 10 + (–9) = 13 +$ ____ (Addition Property)
   5. $3 =$ ____ (Reflexive Property)

IV. Questions/Points to Ponder

Finding solutions to equations in one variable using the properties of equality.
Solving an equation means finding the values of the unknown (such as x) so that the equation becomes true. Although you may solve equations using Guess and Check, a more systematic way is to use the properties of equality as the following examples show.

Example 1. Solve $x – 4 = 8$.

Solution	$x – 4 = 8$	Given
	$x – 4 \mathbf{+ 4} = 8 \mathbf{+ 4}$	APE (Added 4 to both sides)
	$x = 12$	

Checking the solution is a good routine after solving equations. The Substitution Property of Equality can help. This is a good practice for you to check mentally.

$x = 12$ $\quad\quad$ $x – 4 = 8$

$12 – 4 = 8$

$8 = 8$

Since 8 = 8 is true, then the $x = 12$ is a correct solution to the equation.

Example 2. Solve $x + 3 = 5$.

Solution	$x + 3 = 5$	Given
	$x + 3 \mathbf{+ (–3)} = 5 \mathbf{+ (–3)}$	APE (Added –3 to both sides)
	$x = 2$	

Example 3. Solve $3x = 75$.

Solution	$3x = 75$	Given
	$3x \cdot \left(\frac{1}{3}\right) = 75 \cdot \left(\frac{1}{3}\right)$	MPE (Multiplied $\frac{1}{3}$ to both sides)
	$x = 25$	

Note also that multiplying $\frac{1}{3}$ to both sides of the equation is the same as dividing by 3, so the following solution may also be used:

$3x = 75$	Given
$\frac{3x}{3} = \frac{75}{3}$	MPE (Divided both sides of the equation by 3)
$x = 25$	

In Examples 1-3, we saw how the properties of equality may be used to solve an equation and to check the answer. Specifically, the properties were used to "isolate" *x*, or make one side of the equation contain only *x*.

In the next examples, there is an *x* on both sides of the equation. To solve these types of equations, we will use the properties of equality so that all the *x*'s will be on one side of the equation only, while the constant terms are on the other side.

Example 4. Solve $4x + 7 = x - 8$.

Solution

$4x + 7 = x - 8$	Given
$4x + 7 \mathbf{+ (-7)} = x - 8 \mathbf{+ (-7)}$	APE
$4x = x - 15$	
$4x \mathbf{+ (-x)} = x - 15 \mathbf{+ (-x)}$	APE
$3x = -15$	
$\frac{3x}{\mathbf{3}} = \frac{-15}{\mathbf{3}}$	MPE (Multiplied by $\frac{1}{3}$)
$x = -5$	

Example 5. Solve $\frac{x}{3} + \frac{x-2}{6} = 4$.

Solution

$\frac{x}{3} + \frac{x-2}{6} = 4$	Given
$\left(\frac{x}{3} + \frac{x-2}{6}\right) \cdot \mathbf{6} = 4 \cdot \mathbf{6}$	MPE (Multiplied by the LCD: 6)
$2x + (x - 2) = 24$	
$2x + x - 2 = 24$	
$3x - 2 \mathbf{+ 2} = 24 \mathbf{+ 2}$	APE
$3x = 26$	
$\frac{3x}{\mathbf{3}} = \frac{26}{\mathbf{3}}$	MPE (Multiplied by $\frac{1}{3}$)
$x = \frac{26}{3}$	

POINT TO REMEMBER:
In solving linear equations, it is usually helpful to use the properties of equality to combine all terms involving *x* on one side of the equation, and all constant terms on the other side.

V. Exercises:

Solve the following equations, and include all your solutions on your paper.

1. $-6y - 4 = 16$
2. $3x + 4 = 5x - 2$
3. $x - 4 - 4x = 6x + 9 - 8x$
4. $5x - 4(x - 6) = -11$
5. $4(2a + 2.5) - 3(4a - 1) = 5(4a - 7)$

VI. Questions/Points to Ponder

To solve the equation $-14 = 3a - 2$, a student gave the solution below. Read the solution and answer the following questions.

$$\begin{aligned}
-14 &= 3a - 2 \\
-14 + 2 &= 3a - 2 + 2 \\
-12 &= 3a \\
-12 + (-3a) &= 3a + (-3a) \\
-12 - 3a &= 0 \\
-12 - 3a + 12 &= 0 + 12 \\
\frac{-3a}{-3} &= \frac{12}{-3} \\
a &= -4
\end{aligned}$$

1) Is this a correct solution?

2) What suggestions would you have in terms of shortening the method used to solve the equation?

Do equations always have exactly one solution? Solve the following equations and answer the questions.

A) $3x + 5 = 3(x - 2)$

Guide Questions

1) Did you find the value of the unknown?
2) By guess and check, can you think of the solution?
3) This is an equation that has no solution or a null set, can you explain why?
4) Give another equation that has no solution and prove it.

B) $2(x - 5) = 3(x + 2) - x - 16$

Guide Questions

1) Did you find the value of the unknown?
2) Think of 2 or more numbers to replace the variable x and evaluate, what do you notice?
3) This is an equation that has many or infinite solutions, can you explain why?
4) Give another equation that has many or infinite solution and prove it.

C) Are the equations $7 = 9x - 4$ and $9x - 4 = 7$ equivalent equations; that is, do they have the same solution? Defend your answer.

VII. Questions/Points to Ponder

Solving word problems involving equations in one variable. The following is a list of suggestions when solving word problems.

1. Read the problem cautiously. Make sure that you understand the meanings of the words used. Be alert for any technical terms used in the statement of the problem.
2. Read the problem twice or thrice to get an overview of the situation being described.
3. Draw a figure, a diagram, a chart or a table that may help in analyzing the problem.
4. Select a meaningful variable to represent an unknown quantity in the problem (perhaps t, if time is an unknown quantity) and represent any other unknowns in terms of that variable (since the problems are represented by equations in one variable).
5. Look for a guideline that you can use to set up an equation. A guideline might be a formula, such as *distance equals rate times time*, or a statement of a relationship, such as “The sum of the two numbers is 28.”
6. Form an equation that contains the variable and that translates the conditions of the guideline from verbal sentences to equations.
7. Solve the equation, and use the solution to determine other facts required to be solved.
8. Check answers to the **original statement of the problem and not on the equation formulated.**

Example 1. **NUMBER PROBLEM**

Find five consecutive odd integers whose sum is 55.

Solution Let x = 1st odd integer

$x + 2 =$ 2nd odd integer
$x + 4 =$ 3rd odd integer
$x + 6 =$ 4th odd integer
$x + 8 =$ 5th odd integer

$$x + (x + 2) + (x + 4) + (x + 6) + (x + 8) = 55$$
$$5x + 20 = 55$$
$$5x + 20 + (-20) = 55 + (-20)$$
$$\frac{5x}{5} = \frac{35}{5}$$

$x = 7$ The 1st odd integer
$x + 2 = 7 + 2 = 9$ 2nd odd integer
$x + 4 = 7 + 4 = 11$ 3rd odd integer
$x + 6 = 7 + 6 = 13$ 4th odd integer
$x + 8 = 7 + 8 = 15$ 5th odd integer

The five consecutive odd integers are 7, 9, 11, 13, and 15. We can check that the answers are correct if we observe that the sum of these integers is 55, as required by the problem.

Example 2. **AGE PROBLEM**

Margie is 3 times older than Lilet. In 15 years, the sum of their ages is 38 years. Find their present ages.

Representation:

	Age now	**In 15 years**
Lilet	x	x + 15
Margie	3x	3x + 15

In 15 years, the sum of their ages is 38 years.

Equation: $(x + 15) + (3x + 15) = 38$

Solution:

$$\begin{aligned} 4x + 30 &= 38 \\ 4x &= 38 - 30 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

Answer: Lilet’s age now is 2 while, Margie’s age now is 3(2) or 6.

Checking: Margie is 6 which is 3 times older than Lilet who’s only 2 years old. In 15 years, their ages will be 21 and 17. The sum of these ages is 21 + 17 = 38.

VIII. Exercises:

1. The sum of five consecutive integers is 0. Find the integers.
2. The sum of four consecutive even integers is 2 more than five times the first integer. Find the smallest integer.
3. Find the largest of three consecutive even integers when six times the first integer is equal to five times the middle integer.
4. Find three consecutive even integers such that three times the first equals the sum of the other two.
5. Five times an odd integer plus three times the next odd integer equals 62. Find the first odd integer.
6. Al's father is 45. He is 15 years older than twice Al's age. How old is Al?
7. Karen is twice as old as Lori. Three years from now, the sum of their ages will be 42. How old is Karen?
8. John is 6 years older than his brother. He will be twice as old as his brother in 4 years. How old is John now?
9. Carol is five times as old as her brother. She will be three times as old as her brother in two years. How old is Carol now?
10. Jeff is 10 years old and his brother is 2 years old. In how many years will Jeff be twice as old as his brother?

IX. Activity

Solution Papers (Individual Transfer Activity)

Directions: Your teacher will provide two word problems. For each problem, write your solution using the format below. The system that will be used to check your solution paper is provided on the next page.

Name:	Date Submitted:
Year and Section:	Score:
YOUR OWN TITLE FOR THE PROBLEM:	
Problem: ____ ____	
Representation:	
Solution: Conclusion:	
Checking:	

System for Checking Your Solution

	Title	Correctness/ Completeness	Neatness
3 (Exemplary)	The display contains a title that **clearly and specifically** tells what the data shows.	**All data is accurately** represented on the graph. All parts are complete.	The solution paper is very **neat and easy to read.**
2 (Proficient)	The display contains a title that **generally** tells what the data shows.	All parts are complete. Data representation contains **minor errors.**	The solution paper is **generally** neat and readable.
1 (Revision Needed)	The title **does not reflect** what the data shows.	All parts are complete. However, the data is not accurately represented, contains **major errors.** **Or** Some parts are missing and there are minor errors.	The solution paper is **sloppy and difficult** to read.
0 (No Credit)	The title is **missing**.	Some parts and data are **missing**.	The display is a **total mess**.

Summary

This lesson presented the procedure for solving linear equations in one variable by using the properties of equality. To solve linear equations, use the properties of equality to isolate the variable (or *x*) to one side of the equation.

In this lesson, you also learned to solve word problems involving linear equations in one variable. To solve word problems, define the variable as the unknown in the problem and translate the word problem to a mathematical equation. Solve the resulting equation.

Lesson 28: Solving Linear Inequalities Algebraically

Time: 2 hours

Pre-requisite Concepts: Definition of Inequalities, Operation on Integers, Order of Real Numbers

About the Lesson: This lesson discusses the properties of inequality and how these may be used to solve linear inequalities.

Objectives:

In this lesson, you are expected to:

1. State and apply the different properties of inequality;
2. Solve linear inequalities in one variable algebraically; and
3. Solve problems that use first-degree inequality in one variable.

Lesson Proper:

I. Activity

A. Classify each statement as true or false and explain your answer. (You may give examples to justify your answer.)
   1. Given any two real numbers x and y, exactly one of the following statements is true: $x > y$ or $x < y$.
   2. Given any three real numbers a, b, and c. If $a < b$ and $b < c$, then $a < c$.
   3. From the statement "10 > 3", if a positive number is added to both sides of the inequality, the resulting inequality is correct.
   4. From the statement "–12 < –2", if a negative number is added to both sides of the inequality, the resulting inequality is correct.

B. Answer the following questions. Think carefully and multiply several values before giving your answer.
   1. If both sides of the inequality 2 < 5 are multiplied by a non-zero number, will the resulting inequality be true or false?
   2. If both sides of the inequality –3 < 7 are multiplied by a non-zero number, will the resulting inequality be true or false?

II. Questions/Points to Ponder

Properties of Inequalities

The following are the properties of inequality. These will be helpful in finding the solution set of linear inequalities in one variable.

1. Trichotomy Property

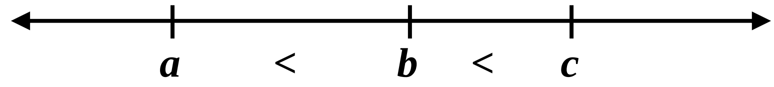
For any number a and b, one and only one of the following is true: $a < b$, $a = b$, or $a > b$.

This property may be obvious, but it draws our attention to this fact so that we can recall it easily next time.

2. Transitive Property of Inequality

For any numbers *a*, *b* and *c*, (a) if $a < b$ and $b < c$, then $a < c$, and
(b) if $a > b$ and $b > c$, then $a > c$.

The transitive property can be visualized using the number line:

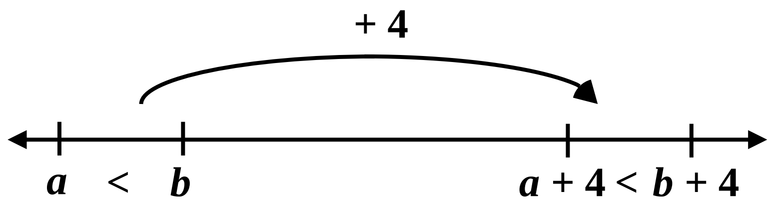


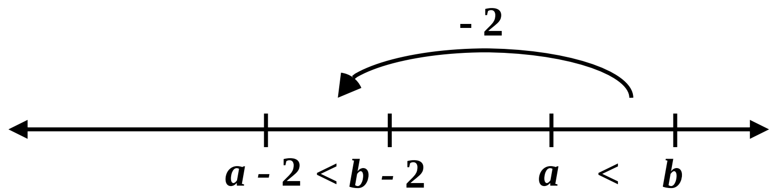
If *a* is to the left of *b*, and *b* is to the left of *c*, then *a* is to the left of *c*.

3. Addition Property of Inequality (API)

For all real numbers *a*, *b* and *c*: (a) if $a < b$, then $a + c < b + c$, and
(b) if $a > b$, then $a + c > b + c.$

Observe that adding the same number to both *a* and *b* **will not** change the inequality. Note that this is true whether we add a **positive** or **negative** number to both sides of the inequality. This property can also be visualized using the number line:



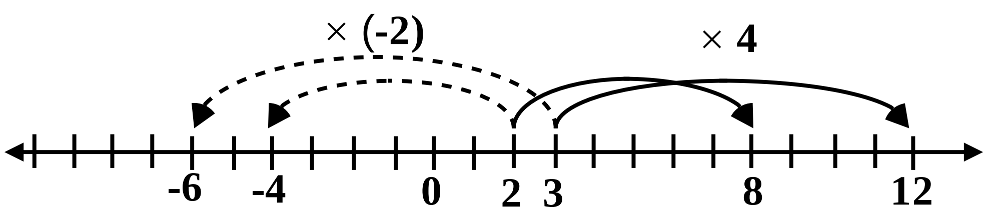


4. Multiplication Property of Inequality (MPI)

For all real numbers *a*, *b* and *c*, then all the following are true:

(a) if $c > 0$ and $a < b$, then $ac < bc$;
(b) if $c > 0$ and $a > b$, then $ac > bc$.
(c) if $c < 0$ and $a < b$, then $ac > bc$;
(d) if $c < 0$ and $a > b$, then $ac < bc$.

Observe that multiplying a **positive** number to both sides of an inequality **does not** change the inequality. However, multiplying a **negative** number to both sides of an inequality **reverses** the inequality. Some applications of this property can be visualized using a number line:



In the number line, it can be seen that if $2 < 3$, then $2(4) < 3(4)$, but $2(-2)$ **>** $3(-2)$.

POINTS TO REMEMBER:

- **Subtracting numbers.** The API also covers subtraction because subtracting a number is the same as adding its negative.
- **Dividing numbers.** The MPI also covers division because dividing by a number is the same as multiplying by its reciprocal.
- **Do not multiply (or divide) by a variable.** The MPI shows that the direction of the inequality depends on whether the number multiplied is positive or negative. However, a variable may take on positive or negative values. Thus, it would not be possible to determine whether the direction of the inequality will be retained not.

III. Exercises

A. Multiple-Choice. Choose the letter of the best answer.

1. What property of inequality is used in the statement "If $m > 7$ and $7 > n$, then $m > n$"?
   A. Trichotomy Property C. Transitive Property of Inequality
   B. Addition Property of Inequality D. Multiplication Property of Inequality
2. If $c > d$ and $p < 0$, then $cp \underline{?} dp$.
   A. $<$ B. $>$ C. $=$ D. Cannot be determined
3. If r and t are real numbers and $r < t$, which one of the following **must** be true?
   A. $-r < -t$ B. $-r > -t$ C. $r < -t$ D. $-r > t$
4. If $w < 0$ and $a + w > c + w$, then what is the relationship between a and c?
   A. $a > c$ B. $a = c$
   C. $a < c$ D. The relationship cannot be determined.
5. If $f < 0$ and $g > 0$, then which of the following is true?
   A. $f + g < 0$ C. $f + g > 0$
   B. $f + g = 0$ D. The relationship cannot be determined.

B. Fill in the blanks by identifying the property of inequality used in each of the following:

1. $x + 11 \geq 23$ Given
   $x + 11 + (-11) \geq 23 + (-11)$ ____________
   $x \geq 12$

2. $5x < -15$ Given
   $(5x)\frac{1}{5} < (-15)\frac{1}{5}$ ____________
   $x < -3$

3. $3x - 7 > 14$ Given
   $3x - 7 + 7 > 14 + 7$ ____________
   $(3x)\frac{1}{3} > (21)\frac{1}{3}$ ____________
   $x > 7$

IV. Activity

Answer each exercise below. After completing all the exercises, compare your work with a partner and discuss.

From the given replacement set, find the solution set of the following inequalities.

1. $2x + 5 > 7$; $\{-6, -3, 4, 8, 10\}$
2. $5x + 4 < -11$; $\{-7, -5, -2, 0\}$
3. $3x - 7 \geq 2$; $\{-2, 0, 3, 6\}$
4. $2x \leq 3x - 1$; $\{-5, -3, -1, 1, 3\}$
5. $11x + 1 < 9x + 3$; $\{-7, -3, 0, 3, 5\}$

Answer the following exercises in groups of five.
What number/expression must be placed in the box to make each statement correct?

1.

$x - 20 < -12$	Given
$x - 20 + \square < -12 + \square$	API
$x < 8$	

2.

$-7x \geq 49$	Given
$(-7x)(\square) \geq (49)(\square)$	MPI
$x \leq -7$	

3.

$\frac{x}{4} - 8 > 3$	Given
$\frac{x}{4} - 8 + \square > 3 + \square$	API
$\frac{x}{4} > 11$	
$\square\left(\frac{x}{4}\right) > (3)\,\square$	MPI
$x > 12$	

4.

$13x + 4 < -5 + 10x$	Given
$13x + 4 + \square < -5 + 10x + \square$	API
$3x + 4 < -5$	
$3x + 4 + \square < -5 + \square$	API
$3x < -9$	
$(\square)3x < (-9)(\square)$	MPI
$x < -3$	

POINTS TO REMEMBER:

- The last statement in each item in the preceding set of exercises is the solution set of the given inequality. For example, in #4, the solution to $13x + 4 < -5 + 10x$ consists of all numbers less than -3 (or $x < -3$). This solution represents all numbers that make the inequality true.
- The solution can be written using set notation as $\{x \mid x < -3\}$. This is read as the "set of all numbers x such that x is less than -3".
- To solve linear inequalities in one variable, isolate the variable that you are solving for on one side of the inequality by applying the properties of inequality.

V. Questions/Points to Ponder

Observe how the properties of inequality may be used to find the solution set of linear inequalities:

1.
$$b + 14 \geq 17$$
$$b + \cancel{14} - \cancel{14} \geq 17 - 14$$
$$b \geq 3$$
Solution Set: $\{b \mid b \geq 3\}$

2.
$$4t - 17 < 51$$
$$4t - 17 + 17 < 51 + 17$$
$$4t < 68$$
$$\frac{4t}{4} < \frac{68}{4}$$
$$t < 17$$
Solution Set: $\{t \mid t < 17\}$

3.
$$2r - 32 \geq 4r + 12$$
$$2r - 32 - 4r \geq 4r + 12 - 4r$$
$$-2r - 32 \geq 12$$
$$-2r - 32 + 32 \geq 12 + 32$$
$$-2r \geq 44$$
$$\frac{-2r}{-2} \geq \frac{44}{-2}$$
$$r \leq -22$$
Solution Set: $\{r \mid r \leq -22\}$

VI. Exercises

Find the solution set of the following inequalities.

1. $b - 19 \leq 15$
2. $9k \leq -27$
3. $-2p > 32$
4. $3r - 5 \geq 4$
5. $2(1 + 5x) < 22$
6. $3w + 10 > 5w + 24$
7. $12x - 40 \geq 11x - 50$
8. $7y + 8 < 17 + 4y$
9. $h - 9 < 2(h - 5)$
10. $10u + 3 - 5u > -18 - 2u$

VII. Questions/Points to Ponder

Match the verbal sentences in column A with the corresponding mathematical statements in column B.

COLUMN A	COLUMN B
1) x is less than or equal to 28.	a) $2x < 28$
2) Two more than x is greater than 28.	b) $x + 2 \geq 28$
3) The sum of a number x and 2 is at least 28.	c) $x + 2 > 28$
4) Twice a number x is less than 28.	d) $x \leq 28$
5) Two less than a number x is at most 28.	e) $x - 2 \leq 28$

Being familiar with translating between mathematical and English phrases will help us to solve word problems, as the following discussion will show.

SOLVING PROBLEMS INVOLVING FIRST-DEGREE INEQUALITY

There are problems in real life that require several answers. Those problems use the concept of inequality. Here are some points to remember when solving word problems that use inequality.

POINTS TO REMEMBER:

- ✓ Read and understand the problem carefully.
- ✓ Represent the unknowns using variables.
- ✓ Formulate an inequality.
- ✓ Solve the inequality formulated.
- ✓ Check or justify your answer.

Example 1. Keith has P5,000.00 in a savings account at the beginning of the summer. He wants to have at least P2,000.00 in the account by the end of the summer. He withdraws P250.00 each week for food and transportation. How many weeks can Keith withdraw money from his account?

Solution:

Step 1: Let w be the number of weeks Keith can withdraw money.

Step 2:

50000	–	250w	$\geq$	2000
↓	↓	↓	↓	↓
amount at the beginning of the summer	withdraw	250 each week	at least	amount at the end of the summer

Step 3:

$$50000 - 250w \geq 2000$$
$$-250w \geq 2000 - 5000$$
$$-250\,w \geq -3000$$
$$w \leq 12$$

Therefore, Keith can withdraw money from his account not more than 12 weeks. We can check our answer as follows. If Keith withdraws P250 per month for 12 months, then the total money withdrawn is P3000. Since he started with P5000, then he will still have P2000 at the end of 12 months.

VIII. Exercises

Solve the following problems on linear inequalities.

1. Kevin wants to buy some pencils at a price of P4.50 each. He does not want to spend more than P55.00. What is the greatest number of pencils can Kevin buy?
2. In a pair of consecutive even integers, five times the smaller is less than four times the greater. Find the largest pair of integers satisfying the given condition.

Summary

In this lesson, you learned about the different properties of linear inequality and the process of solving linear inequalities.

- Many simple inequalities can be solved by adding, subtracting, multiplying or dividing both sides until you are left with the variable on its own.
- The direction of the inequality can change when:
  - Multiplying or dividing both sides by a **negative** number
  - Swapping left and right hand sides
- Do not multiply or divide by a **variable** (unless you know it is always positive or always negative).
- While the procedure for solving linear inequalities is similar to that for solving linear equations, the solution to a linear inequality in one variable usually consists of a range of values rather than a single value.

Lesson 29: Solving Absolute Value Equations and Inequalities

Time: 2.5 hours

Pre-requisite Concepts: Properties of Equations and Inequalities, Solving Linear Equations, Solving Linear Inequalities

About the Lesson: This lesson discusses solutions to linear equations and inequalities that involve absolute value.

Objectives:

In this lesson, the students are expected to:

1. solve absolute value equations;
2. solve absolute value inequalities; and
3. solve problems involving absolute value.

Lesson Proper

I. Activity

Previously, we learned that the absolute value of a number x (denoted by $|x|$) is the distance of the number from zero on the number line. The absolute value of zero is zero. The absolute value of a positive number is itself. The absolute value of a negative number is its opposite or positive counterpart.

Examples are:

$|0| = 0$ $|4| = 4$ $|-12| = 12$ $|7 - 2| = 5$ $|2 - 7| = 5$

Is it true that the absolute value of any number can **never** be negative? Why or why not?

II. Questions/Points to Ponder

By guess-and-check, identify the value/s of the variable that will make each equation TRUE.

1) $|a| = 11$
2) $|m| = 28$
3) $|r| = \frac{12}{17}$
4) $|y| + 1 = 3$
5) $|p| - 1 = 7$
6) $|b| + 2 = 3$
7) $|w - 10| = 1$
8) $\frac{|u|}{2} = 4$
9) $2|x| = 22$
10) $3|c + 1| = 6$

Many absolute value equations are not easy to solve by the guess-and-check method. An easier way may be to use the following procedure.

Step 1: Let the expression on one side of the equation consist only of a single absolute value expression.

Step 2: Is the number on the other side of the equation negative? If it is, then the equation has no solution. (Think, why?) If it is not, then proceed to step 3.

Step 3: If the absolute value of an expression is equal to a positive number, say a, then the expression inside the absolute value can either be a or $-a$ (Again, think, why?). Equate the expression inside the absolute value sign to a and to $-a$, and solve both equations.

Example 1: Solve $|3a - 4| - 9 = 15$.

<table>
<tr><td>Step 1: Let the expression on one side of the equation consist only of a single absolute value expression.</td><td colspan="2">$\lvert 3a - 4\rvert - 11 = 15$
$\lvert 3a - 4\rvert = 26$</td></tr>
<tr><td>Step 2: Is the number on the other side of the equation negative?</td><td colspan="2">No, it’s a positive number, 26, so proceed to step 3</td></tr>
<tr><td>Step 3: To satisfy the equation, the expression inside the absolute value can either be +26 or –26. These correspond to two equations.</td><td>$3a - 4 = 26$</td><td>$3a - 4 = -26$</td></tr>
<tr><td>Step 4: Solve both equations.</td><td>$3a - 4 = 26$
$3a = 30$
$a = 10$</td><td>$3a - 4 = -26$
$3a = -22$
$a = -\frac{22}{3}$</td></tr>
</table>

We can check that these two solutions make the original equation true. If $a = 10$, then $|3a - 4| - 9 = |3(10) - 4| - 9 = 26 - 9 = 15$. Also, if $a = -22/3$, then $|3a - 4| - 9 = |3(-22/3) - 4| - 9 = |-26| - 9 = 15$.

Example 2: Solve $|5x + 4| + 12 = 4$.

<table>
<tr><td>Step 1: Let the expression on one side of the equation consist only of a single absolute value expression.</td><td>$\lvert 5x + 4\rvert + 12 = 4$

$\lvert 5x + 4\rvert = -8$</td></tr>
<tr><td>Step 2: Is the number on the other side of the equation negative?</td><td>Yes, it’s a negative number, –8. There is no solution because $\lvert 5x + 4\rvert$ can never be negative, no matter what we substitute for x.</td></tr>
</table>

Example 3: Solve $|c – 7| = |2c – 2|$.

Step 1: Let the expression on one side of the equation consist only of a single absolute value expression.	Done, because the expression on the left already consists only of a single absolute value expression.	
Step 2: Is the number on the other side of the equation negative?	No, because $|2c – 2|$ is surely **not** negative (the absolute value of a number can never be negative). Proceed to Step 3.	
Step 3: To satisfy the equation, the expression inside the first absolute value, $c – 7$, can either be $+(2c – 2)$ or $–(2c – 2)$. These correspond to two equations. [Notice the similarity to Step 3 of Example 1.]	$c – 7 = +(2c – 2)$	$c – 7 = –(2c – 2)$
Step 4: Solve both equations.	$c – 7 = +(2c – 2)$ $c – 7 = 2c – 2$ $–c – 7 = –2$ $–c = 5$ $c = –5$	$c – 7 = –(2c – 2)$ $c – 7 = –2c + 2$ $3c – 7= 2$ $3c = 9$ $c = 3$

Again, we can check that these two values for *c* satisfy the original equation.

Example 4: Solve $|b + 2| = |b – 3|$

Step 1: Let the expression on one side of the equation consist only of a single absolute value expression.	Done, because the expression on the left already consists only of a single absolute value expression.	
Step 2: Is the number on the other side of the equation negative?	No, because $|b – 3|$ is surely **not** negative (the absolute value of a number can never be negative). Proceed to Step 3.	
Step 3: To satisfy the equation, the expression inside the first absolute value, $b + 2$, can either be equal to $+(b – 3)$ or $–(b – 3)$. These correspond to two equations. [Notice the similarity to Step 3 of Example 1.]	$b + 2 = +(b – 3)$	$b + 2 = –(b – 3)$
Step 4: Solve both equations.	$b + 2 = +(b – 3)$ $b + 2 = b - 3$ $2 = –3$	$b + 2 = –(b – 3)$ $b + 2 = –b +$

	This is false. There is no solution from this equation	3 $2b + 2 = 3$ $2b = 1$ $b = \frac{1}{2}$

Since the original equation is satisfied even if only of the two equations in Step 3 were satisfied, then

this problem has a solution: $b = \frac{1}{2}$. This value of b will make the original equation true.

Example 5: Solve $|x - 4| = |4 - x|$.

Step 1: Let the expression on one side of the equation consist only of a single absolute value expression.	Done, because the expression on the left already consists only of a single absolute value expression.	
Step 2: Is the number on the other side of the equation negative?	No, because $\lvert 4 - x \rvert$ is surely **not** negative (the absolute value of a number can never be negative). Proceed to Step 3.	
Step 3: To satisfy the equation, the expression inside the first absolute value, $x - 4$, can either be equal to $+(4 - x)$ or $-(4 - x)$. These correspond to two equations. [Notice the similarity to Step 3 of Example 1.]	$x - 4 = +(4 - x)$	$x - 4 = -(4 - x)$
Step 4: Solve both equations.	$x - 4 = +4 - x$ $2x - 4 = 4$ $2x = 8$ $x = 4$	$x - 4 = -(4 - x)$ $x - 4 = -4 + x$ $-3 = -3$ This is true no matter what value x is. All real numbers are solutions to this equation

Since the original equation is satisfied even if only of the two equations in Step 3 were satisfied, then the solution to the absolute value equation is the set of all real numbers.

III. Exercises

Solve the following absolute value equations.

1. $\|m\| - 3 = 37$	6. $\|2n - 9\| = \|n + 6\|$
2. $\|2v\| - 4 = 28$	7. $\|5y + 1\| = \|3y - 7\|$
3. $\|5z + 1\| = 21$	8. $\|2t + 3\| = \|2t - 4\|$
4. $\|4x + 2\| - 3 = -7$	9. $\|6w - 2\| = \|6w + 18\|$
5. $\|3a - 8\| + 4 = 11$	10. $\|10 - u\| = \|u - 10\|$

IV. Activity

Absolute Value Inequalities.

You may recall that when solving an absolute value equation, you came up with one, two or more solutions. You may also recall that when solving linear inequalities, it was possible to come up with an interval rather than a single value for the answer.

Now, when solving absolute value inequalities, you are going to combine techniques used for solving absolute value equations as well as first-degree inequalities.

Directions: From the given options, identify which is included in the solution set of the given absolute value inequality. You may have one or more answers in each item.

1. $\|x - 2\| < 3$	a) 5	b) –1	c) 4	d) 0	e) –2
2. $\|x + 4\| \geq 41$	a) –50	b) –20	c) 10	d) 40	e) 50
3. $\left\|\frac{x}{2}\right\| > 9$	a) –22	b) –34	c) 4	d) 18	e) 16
4. $\|2a - 1\| \leq 19$	a) 14	b) 10	c) –12	d) –11	e) –4
5. $2\|u - 3\| < 16$	a) –3	b) –13	c) 7	d) 10	e) 23
6. $\|m + 12\| - 4 > 32$	a) –42	b) –22	c) –2	d) 32	e) 42
7. $\|2z + 1\| + 3 \leq 6$	a) –4	b) –1	c) 3	d) 0	e) 5
8. $\|2r - 3\| - 4 \geq 11$	a) –7	b) –11	c) 7	d) 11	e) 1
9. $\|11 - x\| - 2 > 4$	a) 15	b) 11	c) 2	d) 4	e) 8
10. $\left\|\frac{x}{3} + 1\right\| < 10$	a) –42	b) –36	c) –30	d) –9	e) 21

V. Questions/Points to Ponder

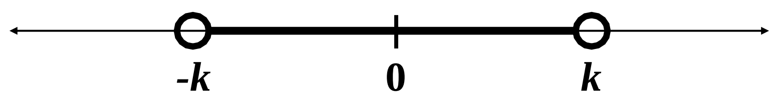
Think about the inequality $|x| < 7$. This means that the expression in the absolute value symbols needs to be less than 7, but it also has to be greater than –7. So answers like 6, 4, 0, –1, as well as many other possibilities will work. With $|x| < 7$, any real number between –7 and 7 will make the inequality true. The solution consists of all numbers satisfying the double inequality $-7 < x < 7$.

Suppose our inequality had been $|x| > 7$. In this case, we want the absolute value of x to be larger than 7, so obviously any number larger than 7 will work (8, 9, 10, etc.). But numbers such as –8, –9, –10 and so on will also work since the absolute value of all those numbers are positive and larger than 7. Thus, the solution or this problem is the set of all x such that $x < -7$ or $x > 7$.

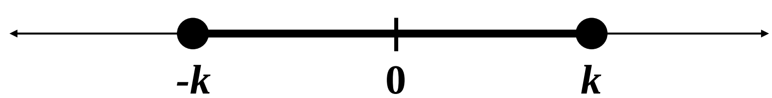
With so many possibilities, is there a systematic way of finding all solutions? The following discussion provides an outline of such a procedure.

In general, an absolute value inequality may be a “less than” or a “greater than” type of inequality (either $|x| < k$ or $|x| > k$). They result in two different solutions, as discussed below.

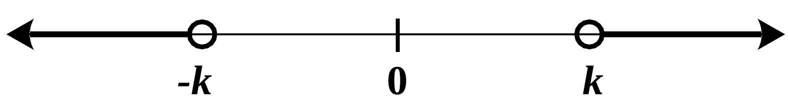
1. Let k be a positive number. Given $|x| < k$, then $-k < x < k$.
   The solution may be represented on the number line. Observe that the solution consists of all numbers whose distance from 0 is less than k.



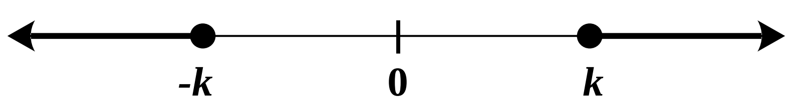
   - If the inequality involves ≤ instead of <, then $\pm k$ will now be part of the solution, which gives $-k \le x \le k$. This solution is represented graphically below.



2. Let k be a positive number. Given $|x| > k$, then $x < -k$ **or** $x > k$.
   The solution may be represented on a number line. Observe that the solution consists of all numbers whose distance from 0 is greater than k.



   - If the inequality involves ≥ instead of >, then $\pm k$ will now be part of the solution, which gives $x \le -k$ **or** $x \ge k$. This solution represented graphically below.



Example 1: Solve $|x – 4| < 18$.

Step 1: This is a "less than" absolute value inequality. Set up a double inequality.	$–18 < x – 4 < 18$
Step 2: Solve the double inequality.	$–18 + 4 < x < 18 + 4$ $–14 < x < 22$

Therefore, the solution of the inequality is $\{x \mid –14 < x < 22\}$. We can check that choosing a number in this set will make the original inequality true. Also, numbers outside this set will not satisfy the original inequality.

Example 2: Solve $|2x + 3| > 13$.

Step 1: This is a "greater than" absolute value inequality. Set up two separate inequalities	$2x + 3 < –13$	$2x + 3 > 13$
Step 2: Solve the two inequalities.	$2x + 3 – 3 < –13 – 3$ $2x < –16$ $x < –8$	$2x + 3 – 3 > 13 – 3$ $2x > 10$ $x > 5$

Therefore, the solution of the inequality is $\{x \mid x < –8 \text{ or } x > 5\}$. This means that all x values less than –8 **<u>or</u>** greater than 5 will satisfy the inequality. By contrast, any number between –8 and 5 (including –8 and 5) will not satisfy the inequality. How do you think will the solution change if the original inequality was ≥ instead of > ?

Example 3: Solve $|3x – 7| – 4 \geq 10$

Step 1: Isolate the absolute value expression on one side.	$\|3x – 7\| \geq 14$	
Step 2: This is a "greater than" absolute value inequality. Set up a two separate inequalities.	$3x – 7 \leq –14$	$3x – 7 \geq 14$
Step 3: Solve the two inequalities	$3x – 7 \leq –14$ $3x + 7 \leq –14 + 7$ $3x \leq –7$ $x \leq -\frac{7}{3}$	$3x – 7 \geq 14$ $3x – 7 + 7 \geq 14 + 7$ $3x \geq 21$ $x \geq 7$

Therefore, the solution of the inequality is $\{x \mid x \leq -\frac{7}{3} \text{ or } x \geq 7\}$.

VI. Exercises

Directions: Solve the following absolute value inequalities and choose the letter of the correct answer from the given choices.

1. What values of *a* satisfy the inequality $|4a + 1| > 5$?

   A. $\{a \mid a < -\frac{3}{2} \text{ or } a > 1\}$ B. $\{a \mid a > -\frac{3}{2} \text{ or } a > 1\}$

   C. $\{a \mid a > -\frac{3}{2} \text{ or } a < 1\}$ D. $\{a \mid a < -\frac{3}{2} \text{ or } a < 1\}$

2. Solve for the values of *y* in the inequality $|y - 20| \leq 4$.

   A. $\{y \mid 16 \geq y \leq 24\}$ B. $\{y \mid 16 \geq y \geq 24\}$

   C. $\{y \mid 16 \leq y \leq 24\}$ D. $\{y \mid 16 \leq y \geq 24\}$

3. Find the solution set of $|b - 7| < 6$.

   A. $\{b \mid -13 < b < 13\}$ B. $\{b \mid 1 < b < 13\}$

   C. $\{b \mid 1 > b > 13\}$ D. $\{b \mid -13 > b > 13\}$

4. Solve for *c*: $|c + 12| + 3 > 17$

   A. $\{c \mid c > -2 \text{ or } c < 2\}$ B. $\{c \mid c > -26 \text{ or } c < 2\}$

   C. $\{c \mid c < -2 \text{ or } c > 2\}$ D. $\{c \mid c < -26 \text{ or } c > 2\}$

5. Solve the absolute value inequality: $|1 - 2w| < 5$

   A. $\{c \mid 3 < c < -2\}$ B. $\{c \mid -3 < c < 2\}$

   C. $\{c \mid 3 > c > -2\}$ D. $\{c \mid -3 > c > 2\}$

VII. Questions/Points to Ponder

Solve the following problems involving absolute value.

1. You need to cut a board to a length of 13 inches. If you can tolerate no more than a 2% relative error, what would be the boundaries of acceptable lengths when you measure the cut board? *(Hint: Let x = actual length, and set up an inequality involving absolute value.)*
2. A manufacturer has a 0.6 oz tolerance for a bottle of salad dressing advertised as 16 oz. Write and solve an absolute value inequality that describes the acceptable volumes for “16 oz” bottles. *(Hint: Let x = actual amount in a bottle, and set up an inequality involving absolute value.)*

Summary

In this lesson you learned how to solve absolute value equations and absolute value inequalities. If *a* is a positive number, then the solution to the absolute value equation $|x| = a$ is $x = a$ or $x = -a$.

There are two types of absolute value inequalities, each corresponding to a different procedure. If $|x| < k$, then $-k < x < k$. If $|x| > k$, then $x < -k$ **or** $x > k$. These principles work for any positive number *k*.