GRADE 7 MATH LEARNING GUIDE

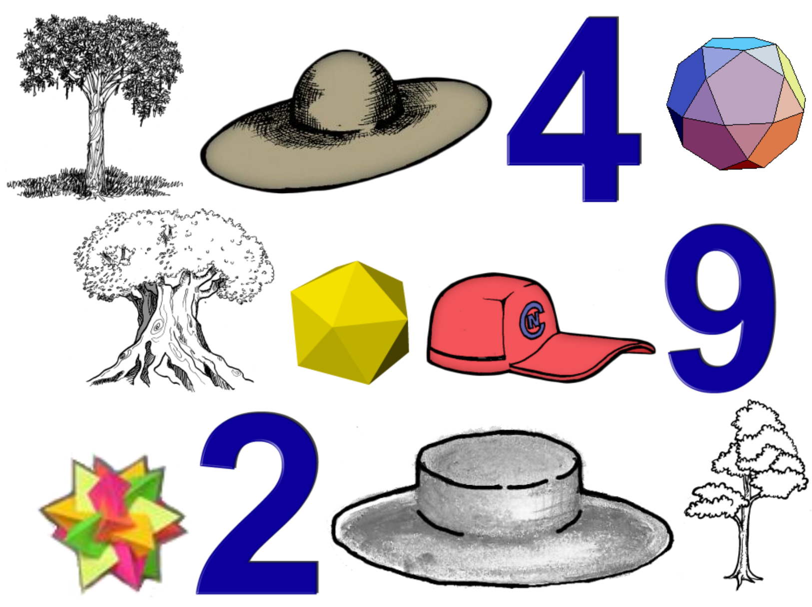
Lesson I: SETS: AN INTRODUCTION **Time: 1.5 hours**
Pre-requisite Concepts: Whole numbers

About the Lesson:
This is an introductory lesson on sets. A clear understanding of the concepts in this lesson will help you easily grasp number properties and enable you to quickly identify multiple solutions involving sets of numbers.

Objectives:
In this lesson, you are expected to:
1. Describe and illustrate
   a. well-defined sets;
   b. subsets;
   c. universal set, and;
   d. the null set.
2. Use Venn Diagrams to represent sets and subsets.

Lesson Proper:
A.
I. Activity

Below are some objects. Group them as you see fit and label each group.



Answer the following questions:
a. How many groups are there?
b. Does each object belong to a group?
c. Is there an object that belongs to more than one group? Which one?

The groups are called sets for as long as the objects in the group share a characteristic and are thus, well defined.

Problem: Consider the set consisting of whole numbers from 1 to 200. Let this be set U. Form smaller sets consisting of elements of U that share a different characteristic. For example, let E be the set of all even numbers from 1 to 200.

Can you form three more such sets? How many elements are there in each of these sets? Do any of these sets have any elements in common?

Did you think of a set with no element?

Important Terms to Remember
The following are terms that you must remember from this point on.

1. A **set** is a well-definedgroup of objects, called **elements** that share a common characteristic. For example, 3 of the objects above belong to the set of head covering or simply hats (ladies hat, baseball cap, hard hat).
2. The set *F* is a **subset** of set *A* if all elements of *F* are also elements of *A*. For example, the even numbers 2, 4 and 12 all belong to the set of whole numbers. Therefore, the even numbers 2, 4, and 12 form a subset of the set of whole numbers. *F* is a **proper subset** of *A* if *F* does not contain all elements of *A*.
3. The **universal set***U* is the set that contains all objects under consideration.
4. The **null set**∅ is an empty set. The null set is a subset of any set.
5. The **cardinality of a set *A*** is the number of elements contained in *A*.

Notations and Symbols
In this section, you will learn some of the notations and symbols pertaining to sets.

1. Uppercase letters will be used to name sets and lowercase letters will be used to refer to any element of a set. For example, let H be the set of all objects on page 1 that cover or protect the head. We write

   H = {ladies hat, baseball cap, hard hat}

   This is the listing or roster method of naming the elements of a set.

   Another way of writing the elements of a set is with the use of a descriptor. This is the rule method. For example,*H* = {x| x covers and protects the head}. This is read as "the set H contains the element x such that x covers and protects the head."

2. The symbol ∅ or { } will be used to refer to an empty set.
3. If *F* is a subset of *A*, then we write $F \subseteq A$. We also say that *A* contains the set *F* and write it as $A \supseteq F$. If *F* is a proper subset of *A*, then we write $F \subset A$.
4. The cardinality of a set *A* is written as n(*A*).

II. Questions to Ponder (Post-Activity Discussion)
Let us answer the questions posed in the opening activity.

1. How many sets are there?
   There is the set of head covers (hats), the set of trees, the set of even numbers, and the set of polyhedra. But, there is also a set of round objects and a set of pointy objects. There are 6 well-defined sets.

2. Does each object belong to a set? *Yes.*

3. Is there an object that belongs to more than one set? Which ones?
   All the hats belong to the set of round objects. The pine trees and two of the polyhedra belong to the set of pointy objects.

III. Exercises

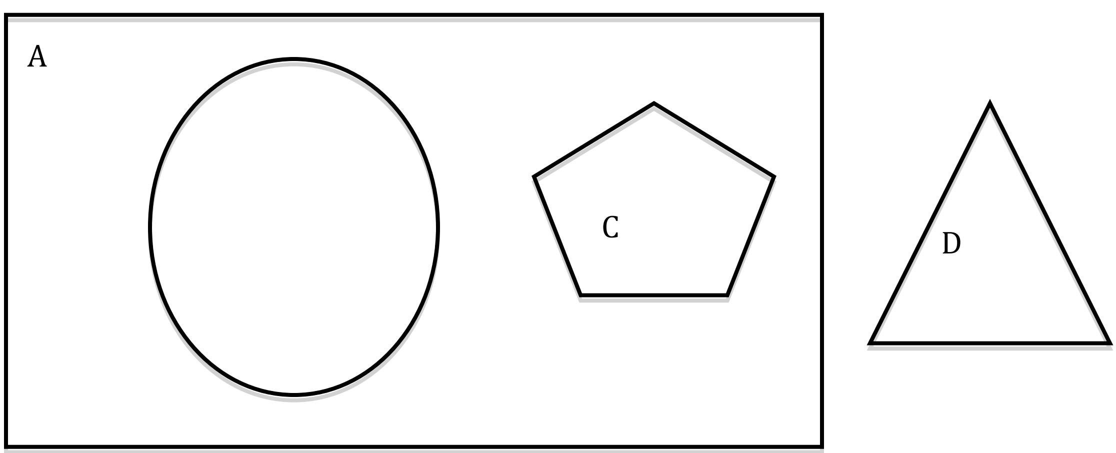
Do the following exercises.

1. Give 3 examples of well-defined sets.
2. Name two subsets of the set of whole numbers using both the listing method and the rule method.
3. Let *B* = [1, 3, 5, 7, 9}. List <u>all</u> the possible subsets of *B*.
4. Answer this question: How many subsets does a set of n elements have?

B. Venn Diagrams

Sets and subsets may be represented using <u>Venn Diagrams</u>. These are diagrams that make use of geometric shapes to show relationships between sets.

Consider the Venn diagram below. Let the universal set U be all the elements in sets A, B, C and D.



Each shape represents a set. Note that although there are no elements shown inside each shape, we can surmise how the sets are related to each other.Notice that set B is inside set A. This indicates that all elements in B are contained in A. The same with set C. Set D, however, is separate from A, B, C. What does it mean?

Exercises

Draw a Venn diagram to show the relationships between the following pairs or groups of sets:

1. E = {2, 4, 8, 16, 32}

F = {2, 32}

2. V is the set of all odd numbers
   W = {5, 15, 25, 35, 45, 55,….}
3. R = {x| x is a factor of 24}
   S = { }
   T = {7, 9, 11}

Summary

In this lesson, you learned about sets, subsets, the universal set, the null set and the cardinality of the set. You also learned to use the Venn diagram to show relationships between sets.

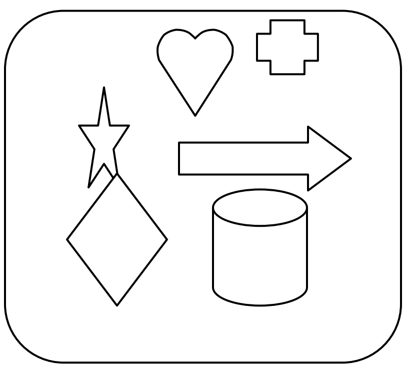
Lesson 2.1: Union and Intersection of Sets **Time: 1.5 hours**

Pre-requisite Concepts: Whole Numbers, definition of sets, Venn diagrams

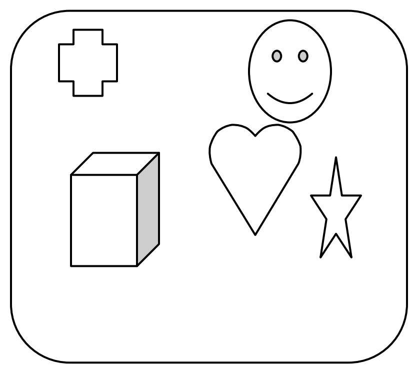
About the Lesson:
After learning some introductory concepts about sets, a lesson on set operations follows. The student will learn how to combine sets (union) and how to determine the elements common to 2 or 3 sets (intersection).

Objectives:
In this lesson, you are expected to:
1. Describe and define
    a. union of sets;
    b. intersection of sets.
2. Perform the set operations
    a. union of sets;
    b. intersection of sets.
3. Use Venn diagrams to represent the union and intersection of sets.

Lesson Proper:
I. Activities

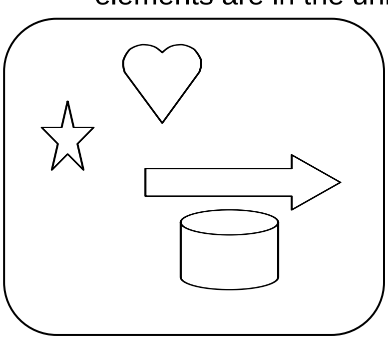


A



B

Answer the following questions:
1. Which of the following shows the union of set A and set B? How many elements are in the union of A and B?



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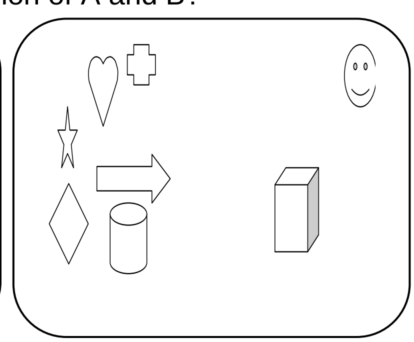

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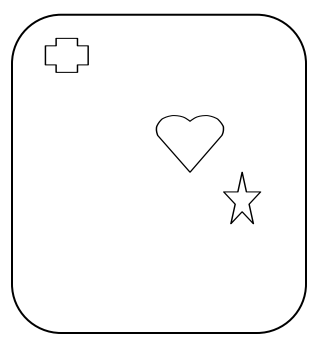
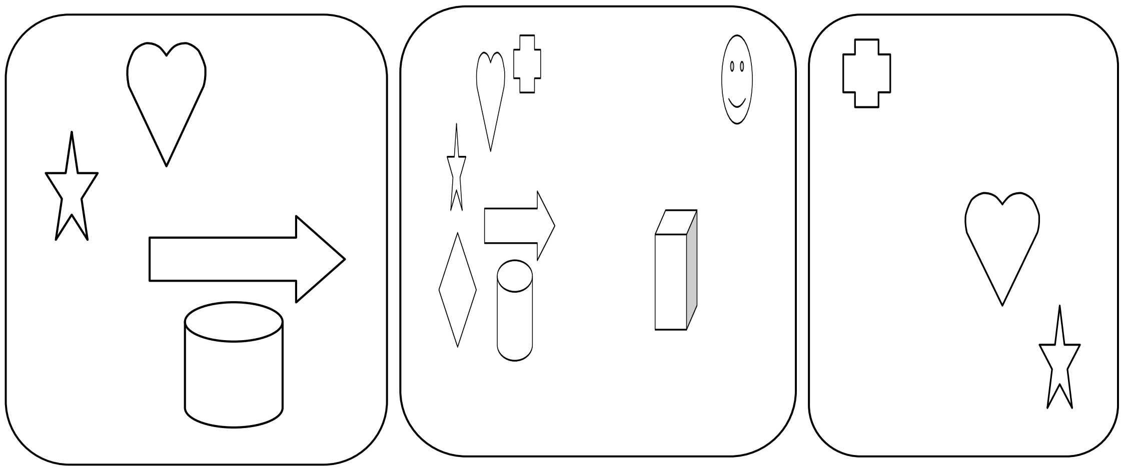


3

2. Which of the following shows the intersection of set A and set B? How many elements are there in the intersection of A and B?



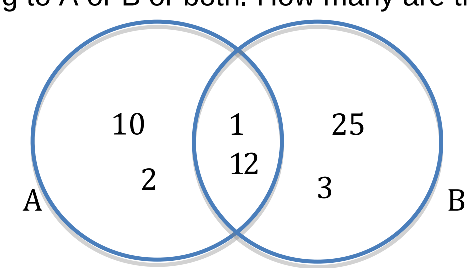
1 **2** **3**

Here's another activity:
Let

$V = \{2x \mid x \in \mathrm{I},\ 1 \leq x \leq 4\}$
$W = \{x^2 \mid x \in \mathrm{I},\ -2 \leq x \leq 2\}$

What elements may be found in the intersection of V and W? How many are there? What elements may be found in the union of V and W? How many are there?

Do you remember how to use Venn Diagrams? Based on the diagram below, (1) determine the elements that belong to both A and B; (2) determine the elements that belong to A or B or both. How many are there in each set?



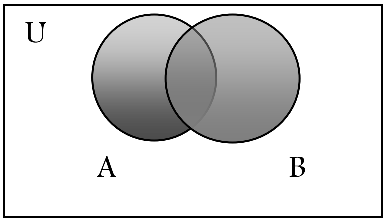
Important Terms/Symbols to Remember
The following are terms that you must remember from this point on.

1. Let A and B be sets. The *union* of the sets A and B, denoted by $A \cup B$, is the set that contains those elements that belong to A, B, or to both.

An element x belongs to the union of the sets A and B if and only if x belongs to A or x belongs to B or to both. This tells us that

$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$

Using the Venn diagram, all of the set of A and of B are shaded to show $A \cup B$.

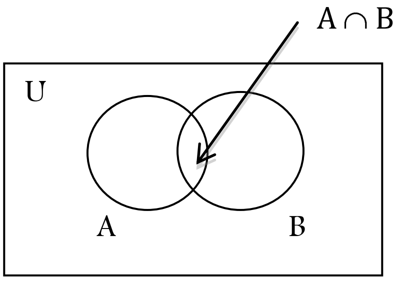


2. Let A and B be sets. The *intersection* of the sets A and B, denoted by $A \cap B$, is the set containing those elements that belong to both A and B.

An element x belongs to the intersection of the sets A and B if and only if x belongs to A and x belongs to B. This tells us that

$A \cap B = \{x \mid x \in A \text{ and } \in B\}$

Using the Venn diagram, the set $A \cap B$ consists of the shared regions of A and B.



Sets whose intersection is an empty set are called *disjoint sets*.

3. The cardinality of the union of two sets is given by the following equation:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions posed in the opening activity.

1. Which of the following shows the union of set A and set B? **Set 2. This is because it contains all the elements that belong to A or B or both. There are 8 elements.**
2. Which of the following shows the intersection of set A and set B? **Set 3. This is because it contains all elements that are in both A and B. There are 3 elements.**

In the second activity:

$V = \{ 2, 4, 6, 8 \}$
$W = \{ 0, 1, 4 \}$

Therefore, $V \cap W = \{ 4 \}$ has 1 element and $V \cup W = \{ 0, 1, 2, 4, 6, 8 \}$ has 6 elements. Note that the element { 4 } is counted only once.

On the Venn Diagram: (1) The set that contains elements that belong to both A and B consists of two elements {1, 12 }; (2) The set that contains elements that belong to A or B or both consists of 6 elements {1, 10, 12, 20, 25, 36 }.

III. Exercises

1. Given sets A and B,

Set A **Students who play the guitar**	**Set B** **Students who play the piano**
Ethan Molina	Mayumi Torres
Chris Clemente	Janis Reyes
Angela Dominguez	Chris Clemente
Mayumi Torres	Ethan Molina
Joanna Cruz	Nathan Santos

determine which of the following shows (a) $A \cup B$; and (b) $A \cap B$?

Set 1	**Set 2**	**Set 3**	**Set 4**
Ethan Molina Chris Clemente Angela Dominguez Mayumi Torres Joanna Cruz	Mayumi Torres Ethan Molina Chris Clemente	Mayumi Torres Janis Reyes Chris Clemente Ethan Molina Nathan Santos	Ethan Molina Chris Clemente Angela Dominguez Mayumi Torres Joanna Cruz Janis Reyes Nathan Santos

2. Do the following exercises. Write your answers on the spaces provided:

$A = \{0, 1, 2, 3, 4\}$ $B = \{0, 2, 4, 6, 8\}$ $C = \{1, 3, 5, 7, 9\}$

Given the sets above, determine the elements and cardinality of:

a. $A \cup B$ = ______________________
b. $A \cup C$ = ______________________
c. $A \cup B \cup C$ = ______________________
d. $A \cap B$ = ______________________
e. $B \cap C$ = ______________________
f. $A \cap B \cap C$ = ______________________
g. $(A \cap B) \cup C$ = ______________________

3. Let $W = \{ x \mid 0 < x < 3 \}$, $Y = \{ x \mid x > 2\}$, and $Z = \{x \mid 0 \leq x \leq 4 \}$. Determine (a) $(W \cup Y) \cap Z$; (b) $W \cap Y \cap Z$.

Summary

In this lesson, you learned the definition of union and intersection of sets. You also learned how use Venn diagram to represent the union and the intersection of sets. You also learned how to determine the elements that belong to the union and intersection of sets.

Lesson 2.2: Complement of a Set **Time: 1.5 hours**

Prerequisite Concepts: sets, universal set, empty set, union and intersection of sets, cardinality of sets, Venn diagrams

About the Lesson:

The complement of a set is an important concept. There will be times when one needs to consider the elements not found in a particular set A. You must know that this is when you need the complement of a set.

Objectives:

In this lesson, you are expected to:
1. Describe and define the complement of a set;
2. Find the complement of a given set;
3. Use Venn diagrams to represent the complement of a set.

Lesson Proper:

I. Problem

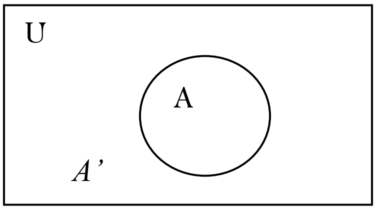
In a population of 8000 students, 2100 are Freshmen, 2000 are Sophomores, 2050 are Juniors and the remaining 1850 are either in their fourth or fifth year in university. A student is selected from the 8000 students and it is not a Sophomore, how many possible choices are there?

Discussion

Definition: The complement of a set A, written as *A'*, is the set of all elements found in the universal set, U, that are not found in set A. The cardinality n (*A'*) is given by

$$n(A') = n(U) - n(A) .$$

Venn diagram:



Examples:

1. Let U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, and A = {0, 2, 4, 6, 8}.
Then the elements of *A'* are the elements from U that are not found in A.
Therefore, *A'* = {1, 3, 5, 7, 9} and n (*A'*) = 5

2. Let U = {1, 2, 3, 4, 5}, A = {2, 4} and B = {1, 5}. Then
$A' = \{1, 3, 5\}$
$B' = \{2, 3, 4\}$
$A' \cup B' = \{1, 2, 3, 4, 5\} = U$

3. Let U = {1, 2, 3, 4, 5, 6, 7, 8}, A = {1, 2, 3, 4} and B = {3, 4, 7, 8}.
Then

$A' = \{5, 6, 7, 8\}$
$B' = \{1, 2, 5, 6\}$
$A' \cap B' = \{5, 6\}$

4. Let U = {1, 3, 5, 7, 9}, A = {5, 7, 9} and B = {1, 5, 7, 9}. Then
   $A \cap B = \{5, 7, 9\}$
   $(A \cap B)' = \{1, 3\}$

5. Let U be the set of whole numbers. If A = {x | x is a whole number and x > 10}, then A' = {x | x is a whole number and $0 \leq x \leq 10$}.

The opening problem asks for how many possible choices there are for a student that was selected and known to be a non-Sophomore. Let U be the set of all students and n (U) = 8000. Let A be the set of all Sophomores then n (A) = 2000. The set A' consists of all students in U that are not Sophomores and n (A') = n (U) – n (A) = 6000. Therefore, there are 6000 possible choices for that selected student.

II. Activity

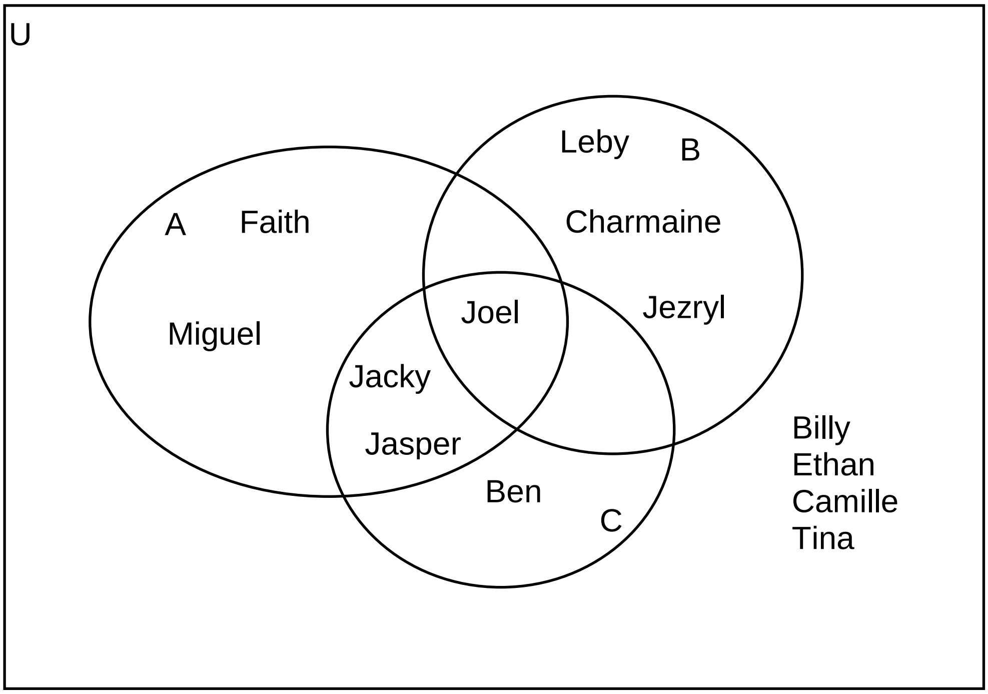
Shown in the table are names of students of a high school class by sets according to the definition of each set.

A **Likes Singing**	**B** **Likes Dancing**	**C** **Likes Acting**	**D** **Don't Like Any**
Jasper Faith Jacky Miguel Joel	Charmaine Leby Joel Jezryl	Jacky Jasper Ben Joel	Billy Ethan Camille Tina

After the survey has been completed, find the following sets.

a. U = ____
b. $A \cup B'$ = ____
c. $A' \cup C$ = ____
d. $(B \cup D)'$ = ____
e. $A' \cap B$ = ____
f. $A' \cap D'$ = ____
g. $(B \cap C)'$ = ____

The easier way to find the elements of the indicated sets is to use a Venn diagram showing the relationships of U, sets A, B, C, and D. Set D does not share any members with A, B, and C. However, these three sets share some members. The Venn diagram below is the correct picture:



Now, it is easier to identify the elements of the required sets.

a. U = {Ben, Billy, Camille, Charmaine, Ethan, Faith, Jacky, Jasper, Jezryl, Joel, Leby, Miguel, Tina}
b. A ∪ *B'* = {Faith, Miguel, Joel, Jacky, Jasper, Ben, Billy, Ethan, Camille, Tina}
c. *A'* ∪ C = {Jasper, Jacky, Joel, Ben, Leby, Charmaine, Jezryl, Billy, Ethan, Camille, Tina}
d. *(B* ∪ *D)'* = {Faith, Miguel, Jacky, Jasper, Ben}
e. *A'* ∩ B = {Leby, Charmaine, Jezryl}
f. *A'* ∩ *D'* = {Leby, Charmaine, Jezryl, Ben}
g. *(B* ∩ *C)'* = {Ben, Billy, Camille, Charmaine, Ethan, Faith, Jacky, Jasper, Jezryl, Leby, Miguel, Tina}

III. Exercises

1. True or False. If your answer is false, give the correct answer.

Let U = the set of the months of the year
X = {March, May, June, July, October}
Y = {January, June, July}
Z = {September, October, November, December}

a. *Z'* = {January, February, March, April, May, June, July, August}

____

b. *X'* ∩ *Y'* = {June, July}

____

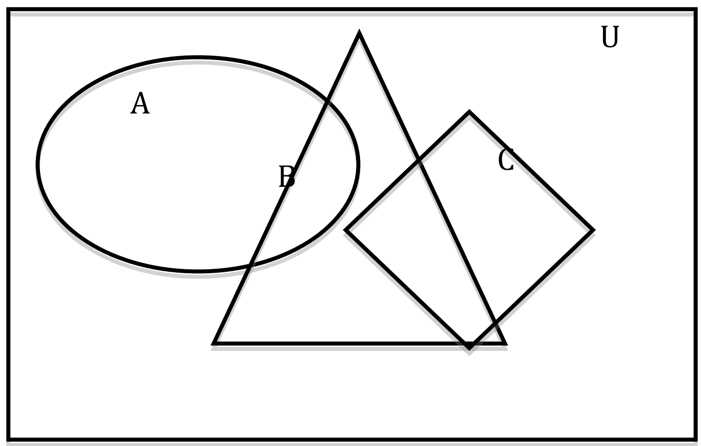
c. *X'* ∪ *Z'* = {January, February, March, April, May, June, July, August, September, November, December}

____

d. *(Y ∪ Z)'* = {February, March, April, May}

____

2. Place the elements in their respective sets in the diagram below based on the following elements assigned to each set:



U = {a, b, c, d, e, f, g, h, i, j}
A' = {a, c, d, e, g, j}
B' = {a, b, d, e, h, i}
C' = {a, b, c, f, h, i, j}

3. Draw a Venn diagram to show the relationships between sets U, X, Y, and Z, given the following information.

- U, the universal set contains set X, set Y, and set Z.
- X ∪ Y ∪ Z = U
- Z is the complement of X.
- Y' includes some elements of X and the set Z

Summary

In this lesson, you learned about the complement of a given set. You learned how to describe and define the complement of a set, and how it relates to the universal set, U and the given set.

Lesson 3: **Problems Involving Sets** **Time: 1 hour**

Prerequisite Concepts: Operations on Sets and Venn Diagrams

About the Lesson:

This is an application of your past lessons about sets. You will appreciate more the concepts and the use of Venn diagrams as you work through the different word problems.

Objectives:

In this lesson, you are expected to:

1. Solve word problems involving sets with the use of Venn diagrams
2. Apply set operations to solve a variety of word problems.

Lesson Proper:

I. Activity

Try solving the following problem:

In a class of 40 students, 17 have ridden an airplane, 28 have ridden a boat. 10 have ridden a train, 12 have ridden both an airplane and a boat. 3 have ridden a train only and 4 have ridden an airplane only. Some students in the class have not ridden any of the three modes of transportation and an equal number have taken all three.

a. How many students have used all three modes of transportation?
b. How many students have taken only the boat?

II. Questions/Points to Ponder (Post-Activity Discussion)

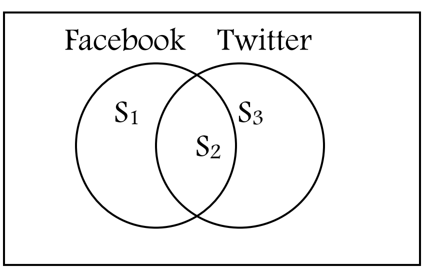
Venn diagrams can be used to solve word problems involving union and intersection of sets. Here are some worked out examples:

1. A group of 25 high school students were asked whether they use either Facebook or Twitter or both. Fifteen of these students use Facebook and twelve use Twitter.
   a. How many use Facebook only?
   b. How many use Twitter only?
   c. How many use both social networking sites?

<u>Solution:</u>

Let S_1 = set of students who use Facebook only
S_2 = set of students who use both social networking sites
S_3 = set of students who use Twitter only

The Venn diagram is shown below



Finding the elements in each region:

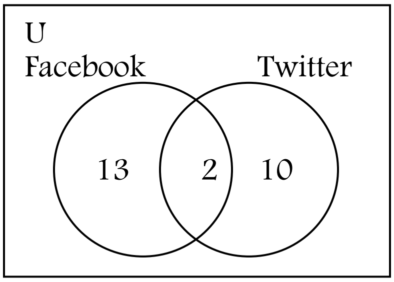
$$\begin{array}{lr} n(S_1) + n(S_2) + n(S_3) & = 25 \\ n(S_1) + n(S_2) & = 15 \\ \hline n(S_3) & = 10 \end{array}$$

But $n(S_2) + n(S_3) = 12$

$$\begin{array}{lr} \hline n(S_2) & = 2 \end{array}$$

$$\begin{array}{lr} n(S_1) + n(S_2) + n(S_3) & = 25 \\ n(S_2) + n(S_3) & = 12 \\ \hline n(S_1) & = 13 \end{array}$$

The number of elements in each region is shown below



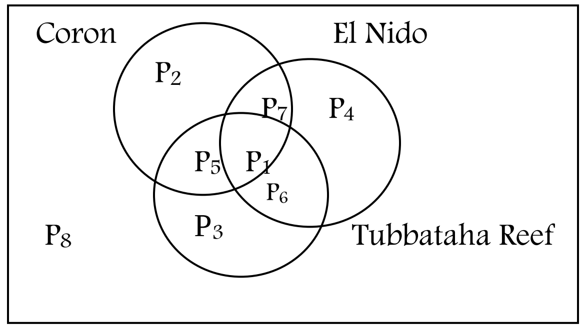
2. A group of 50 students went in a tour in Palawan province. Out of the 50 students, 24 joined the trip to Coron; 18 went to Tubbataha Reef; 20 visited El Nido; 12 made a trip to Coron and Tubbataha Reef; 15 saw Tubbataha Reef and El Nido; 11 made a trip to Coron and El Nido and 10 saw the three tourist spots.
   a. How many of the students went to Coron only?
   b. How many of the students went to Tubbataha Reef only?
   c. How many joined the El Nido trip only?
   d. How many did not go to any of the tourist spots?

Solution:

To solve this problem, let

P_1 = students who saw the three tourist spots
P_2 = those who visited Coron only
P_3 = those who saw Tubbataha Reef only
P_4 = those who joined the El Nido trip only
P_5 = those who visited Coron and Tubbataha Reef only
P_6 = those who joined the Tubbataha Reef and El Nido trip only
P_7 = those who saw Coron and El Nido only
P_8 = those who did not see any of the three tourist spots

Draw the Venn diagram as shown below and identify the region where the students went.



Determine the elements in each region starting from P_1.

P_1 consists of students who went to all 3 tourist spots. Thus, $n(P_1) = 10$.

$P_1 \cup P_5$ consists of students who visited Coron and Tubbataha Reef but this set includes those who also went to El Nido. Therefore, $n(P_5) = 12 – 10 = 2$ students visited Coron and Tubbatha Reef only.

$P_1 \cup P_6$ consists of students who went to El Nido and Tubbataha Reef but this set includes those who also went to Coron. Therefore, $n(P_6) = 15 – 10 = 5$ students visited El Nido and Tubbataha Reef only.

$P_1 \cup P_7$ consists of students who went to Coron and El Nido but this set includes those who also went to Tubbataha Reef. Therefore, $n(P_7) = 11 – 10 = 1$ student visited Coron and El Nido only.

From here, it follows that

$n(P_2) = 24 – n(P_1) – n(P_5) – n(P_7) = 24 – 10 – 2 – 1 =$ *11 students visited Coron only.*

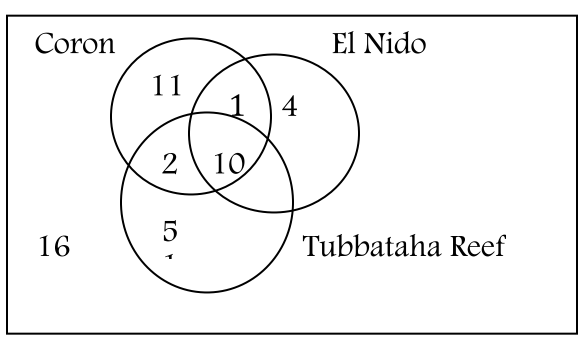
$n(P_3) = 18 – n(P_1) – n(P_5) – n(P_6) = 18 – 10 – 2 – 5 =$ *1 student visited Tubbataha Reef only*

$n(P_4) = 20 – n(P_1) – n(P_6) – n(P_7) = 20 – 10 – 5 – 1 =$ *4 students visited Coron and El Nido only.*

Therefore

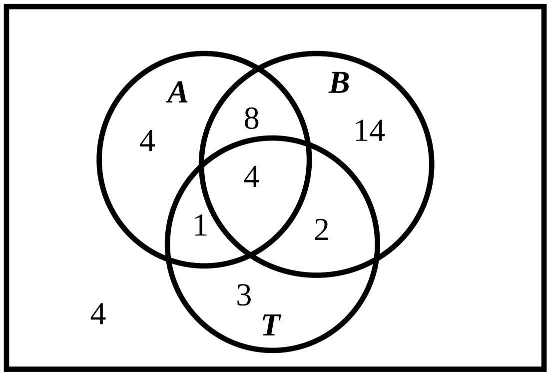
$n(P_8) = 50 – n(P_1) – n(P_2) – n(P_3) – n(P_4) – n(P_5) – n(P_6) – n(P_7) =$ *16 students did not visit any of the three spots.*

The number of elements is shown below.



Now, what about the opening problem?

Solution to the Opening Problem (Activity):

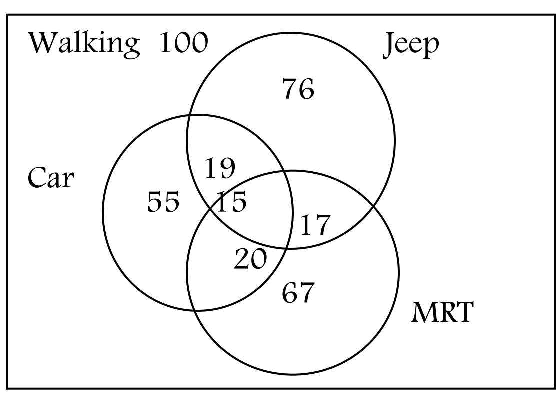


Can you explain the numbers?

III. Exercises

Do the following exercises. Represent the sets and draw a Venn diagram when needed.

1. If **A** is a set, give two subsets of **A**.
2. **(a)** If A and B are finite sets and $A \subset B$, what can you say about the cardinalities of the two sets?
   (b) If the cardinality of A is less than the cardinality of B, does it follow that $A \subset B$?
3. If **A** and **B** have the same cardinality, does it follow that **A = B?** Explain.
4. If $A \subset B$ and $B \subset C$. Does it follow that $A \subset C$? Illustrate your reasoning using a Venn diagram.
5. Among the 70 kids in Barangay Magana, 53 like eating in Jollibee while 42 like eating in McDonalds. How many like eating both in Jollibee and in McDonalds? in Jollibee only? in McDonalds only?
6. The following diagram shows how all the First Year students of Maningning High School go to school.



   a. How many students ride in a car, jeep and the MRT going to their school? ________
   b. How many students ride in both a car and a jeep? ________
   c. How many students ride in both a car and the MRT? ________

d. How many students ride in both a jeep and the MRT? _______

e. How many students go to school
in a car only ______ a jeep only _______
in the MRT only _________ walking _______

f. How many students First Year students of Maningning High School are there? ________

7. The blood-typing system is based on the presence of proteins called antigens in the blood. A person with antigen A has blood type A. A person with antigen B has blood type B, and a person with both antigens A and B has blood type AB. If no antigen is present, the blood type is O. Draw a Venn diagram representing the ABO System of blood typing.

   A protein that coats the red blood cells of some persons was discovered in 1940. A person with the protein is classified as Rh positive (Rh+), and a person whose blood cells lack the protein is Rh negative (Rh–). Draw a Venn diagram illustrating all the blood types in the ABO System with the corresponding Rh classifications.

Summary

In this lesson, you were able to apply what you have learned about sets, the use of a Venn diagram and set operations in solving word problems.

Lesson 4.1: Fundamental Operations on Integers: Addition of Integers
Time: 1 hour

Pre-requisite Concepts: Whole numbers, Exponents, Concept of Integers

About the Lesson: This lesson focuses on addition of integers using different approaches. It is a review of what the students learned in Grade 6.

Objectives:

In this lesson, you are expected to:

1. Add integers using different approaches;
2. Solve word problems involving addition of integers.

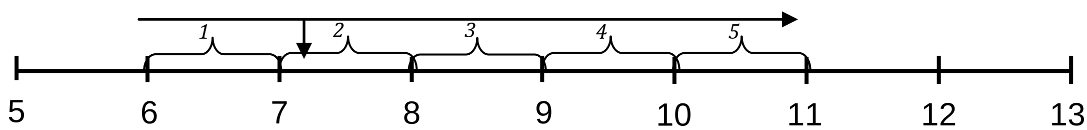
Lesson Proper:

I. Activity

Study the following examples:

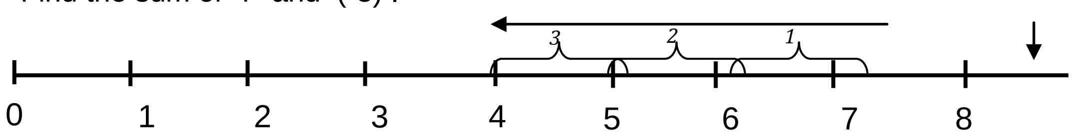
A. Addition Using Number Line

1. Use the number line to find the sum of 6 & 5.



On the number line, start with point 6 and count 5 units to the right. At what point on the number line does it stop ?
It stops at point 11; hence, 6 + 5 = 11.

2. Find the sum of 7 and (-3) .



On the number line, start from 7 and count 3 units going to the left since the sign of 3 is negative.
At which point does it stop?
It stops at point 4; hence, (-3) + (7) = 4.

After the 2 examples, can you now try the next two problems?

a. (-5) + (-4) b. (-8) + (5)

We now have the following generalization:
Adding a positive integer n to m means moving along the real line a distance of n units to the right from m. Adding a negative integer $-n$ to m means moving along the real line a distance of n units to the left from m.

B. Addition Using Signed Tiles

This is another device that can be used to represent integers.The tile $\boxed{+}$ represents integer 1, the tile $\boxed{-}$ represents -1, and the flexible $\boxed{+ \quad -}$ represents 0.

Recall that a number and its negative cancel each other under the operation of addition. This means

$4 + -4 = 0$

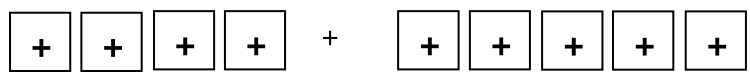
$15 + -15 = 0$

$-29 + 29 = 0$

In general, $n + -n = -n + n = 0$.

Examples:

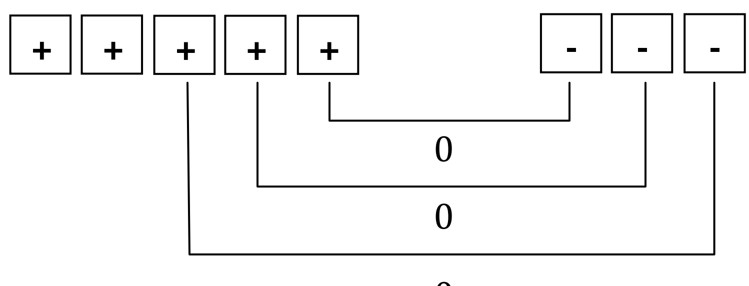
1. 4 + 5 ------→



four (+1) + five (+1)

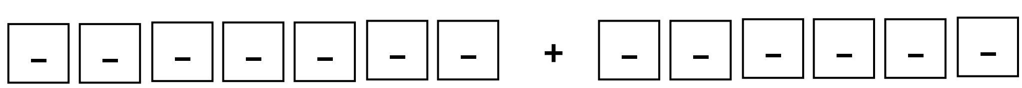
hence, 4 + 5 = 9

2. 5 + (-3) -----→



hence, $5 + -3 = 2 + 3 + -3 = 2 + 0 = 2$

3. $-7 + -6$



hence $-7 + -6 = -13$

Now, try these:

1. (-5) + (-11)
2. (6) + (-9)

II. Questions/ Points to Ponder

Using the above model, we summarize the procedure for adding integers as follows:

1. If the integers have the same sign, just add the positive equivalents of the integers and attach the common sign to the result.

   a. 27 + 30 = + (/27/ + /30/)
      = + (/57/)
      = + 57

   b. (-20) + (-15) = - (/20/ + /15/)
      = - (20 + 15)
      = - (35)
      = - 35

2. If the integers have different signs, get the difference of the positive equivalents of the integers and attach the sign of the larger number to the result.

   a. (38) + (-20)
      Get the difference between 38 and 20: 18
      Since 38 is greater than 20, the sign of the sum is positive.
      Hence $38 + \ -20 \ = 18$

   b. $-42 \ + 16$
      Get the difference between 42 and 16: 26
      Since 42 is greater than 16, the sum will have a negative sign.
      Hence $-42 \ + 16 = -26$

If there are more than two addends in the problem the first step to do is to combine addends with same signs and then get the difference of their sums.

Examples:

1. $-14 + 22 + 8 + -16 = - \ 14 + 16 + \ 22 + 8$
   $= -30 + 30 = 0$

2. $31 + 70 + 9 + \ -155 = \ 31 + 70 + 9 + \ -155$
   $= 110 + \ -155 \ = -45$

III. Exercises

A. Who was the first English mathematician who first used the modern symbol of equality in 1557?

   (To get the answer, compute the sums of the given exercises below. Write the letter of the problem corresponding to the answer found in each box at the bottom).

A 25 + 95	C. (30) + (-20)	R 65 + 75
B 38 + (-15)	D. (110) + (-75)	O (-120) + (-35)

O 45 + (-20)	T. (16) + (-38)	R (165) + (-85)
R (-65) + (-20)	R (-65) + (-40)	E 47 + 98
E (78) + (-15)	E (-75) + (20)	

-105	25	63	23	-85	-22

140	-55	10	-155	80	35	145

B. Addthe following:

1. (18) + (-11) + (3)
2. (-9) + (-19) + (-6)
3. (-4) + (25) + (-15)
4. (50) + (-13) + (-12)
5. (-100) + (48) + (49)

C. Solve the following problems:

1. Mrs. Reyes charged P3,752.00 worth of groceries on her credit card. Find her balance after she made a payment of P2,530.00.
2. In a game, Team Azcals lost 5 yards in one play but gained 7 yards in the next play. What was the actual yardage gain of the team?
3. A vendor gained P50.00 on the first day; lost P28.00 on the second day, and gained P49.00 on the third day. How much profit did the vendor gain in 3 days?
4. Ronnie had PhP2280 in his checking account at the beginning of the month. He wrote checks for PhP450, P1200, and PhP900. He then made a deposit of PhP1000. If at any time during the month the account is overdrawn, a PhP300 service charge is deducted. What was Ronnie's balance at the end of the month?

Summary

In this lesson, you learned how to add integers using two different methods. The number line model is practical for small integers. For larger integers, the signed tiles model provides a more useful tool.

Lesson 4.2: Fundamental Operation on Integers:Subtraction of Integers
Time: 1 hour

Prerequisite Concepts: Whole numbers, Exponents, Concept of Integers, Addition of Integers

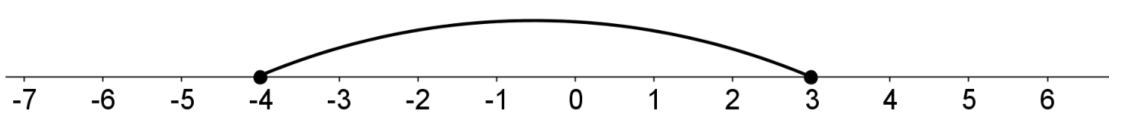
About the Lesson: This lesson focuses on the subtraction of integers using different approaches. It is a review of what the students learned in Grade 6.

Objectives:
In this lesson, you are expected to:
1. Subtract integers using
   a. Number line
   b. Signed tiles
2. Solve problems involving subtraction of integers.

Lesson Proper:

I. Activity
Study the material below.

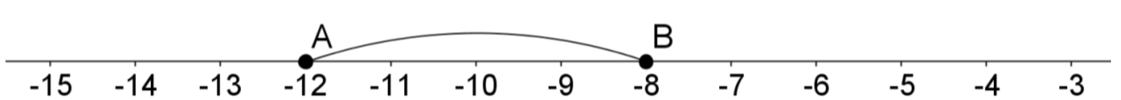
1. *Subtraction as the reverse operation of addition.*
   Recall how subtraction is defined. We have previously defined subtraction as the reverse operation of addition. This means that when we ask "what is 5 minus 2?", we are also asking "what number do we add to 2 in order to get 5?" Using this definition of subtraction, we can deduce how subtraction is done using the number line.



a. Suppose you want to compute $-4\ -3$. You ask "What number must be added to 3 to get -4?

   To get from 3 to -4, you need to move 7 units to the left. This is equivalent to adding -7 to 3. Hence in order to get -4, -7 must be added to 3. Therefore,
$(-4)-3=-7$

b. Compute $(-8)-(-12)$
   What number must be added to -12 to get -8?



To go from -12 to -8, move 4 units to the right, or equivalently, add 4. Therefore,

$(-8) - \ \ -12 \ \ = 4$

2. *Subtraction as the addition of the negative*
   Subtraction is also defined as the addition of the negative of the number. For example, $5 - 3 = 5 + (-3)$. Keeping in mind that n and $-n$ are negatives of each other, we can also have $5 - \ \ -3 \ \ = 5 + 3$. Hence the examples above can be solved as follows:

$$-4 \ \ - 3 = (-4) + \ \ -3 \ \ = -7$$

$$-8 \ \ - \ \ -12 \ \ = (-8) + 12 = 4$$

   This definition of subtraction allows the conversion of a subtraction problem to an addition problem.

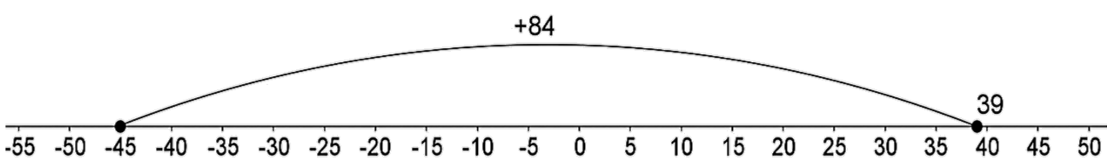
Problem:

Subtract (-45) from 39 using the two definitions of subtraction.

Can you draw your number line?Where do you start numbering it to make the line shorter?

Solution:

1. $39 - (-45)$
   What number must be added to -45 in order to obtain 39?



$$39 - \ \ -45 \ \ = 84$$

2. $39 - \ \ -45 \ \ = 39 + 45 = 84$

II. Questions/Points to Ponder

Rule in Subtracting Integers

In subtracting integers, add the negative of the subtrahend to the minuend,

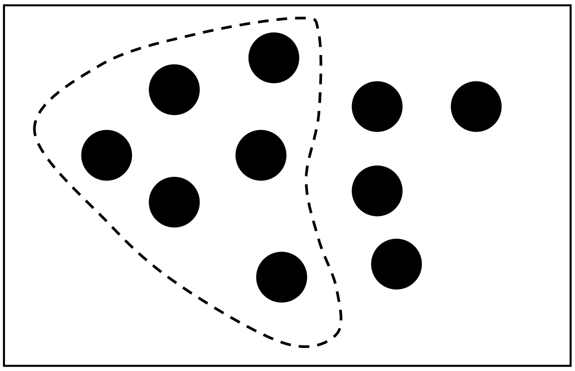
$$a - b = a + \ \ -b$$
$$a - \ \ -b \ \ = a + b$$

Using signed tiles or colored counters

Signed tiles or colored counters can also be used to model subtraction of integers. In this model, the concept of subtraction as "taking away" is utilized.

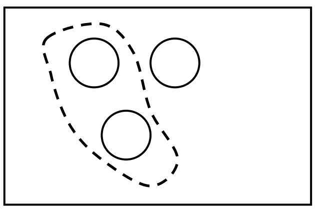
Examples:

1. $10 - 6$ means take away 6 from 10. Hence



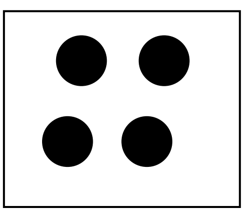
$10 - 6 = 4$

2. $-3 - (-2)$

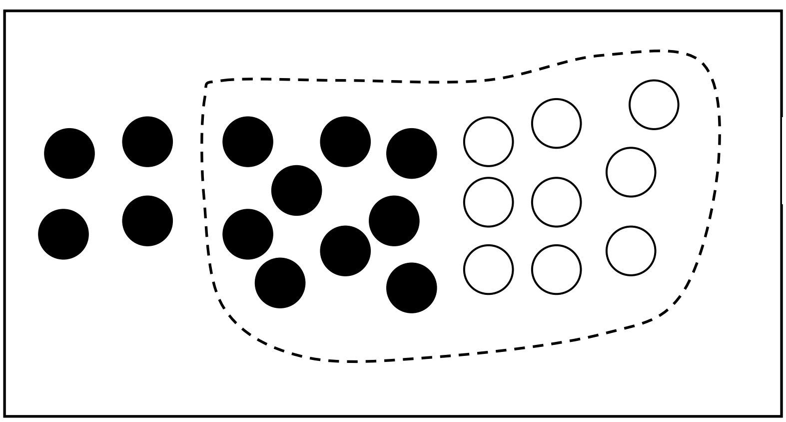


$-3 - (-2) = -1$

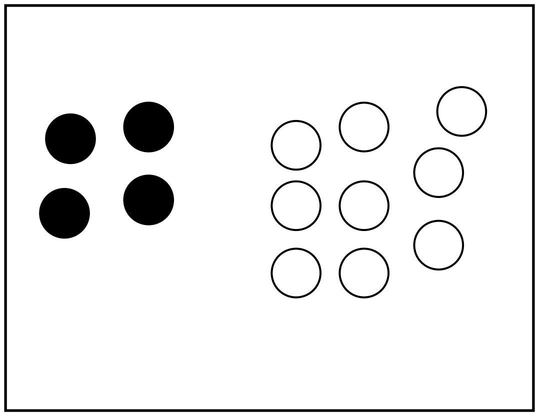
3. $4 - 9$



Since there are not enough counters from which to take away 9, we add 9 black counters and 9 white counters. Remember that these added counters are equivalent to zero.



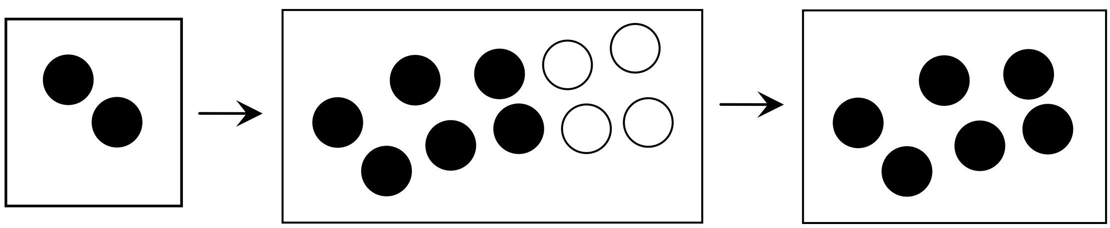
We now take away 9 black counters.



Notice that this configuration is the same configuration for$4 + (-9)$.

We proceed with the addition and obtain the answer -5

4. $2 - (-4)$



Hence $2 - \quad -4 \quad = 6$

The last two examples above illustrate the definition of subtraction as the addition of the negative.
$m - n = m - n + \ n + \ -n \quad = \ m - n + n \ + \ -n \ = m + (-n)$

III. Exercices

A. What is the name of the 4th highest mountain in the world?
(Decode the answer by finding the difference of the following subtraction problems. Write the letter to the answer corresponding to the item in the box provided below:

O Subtract (-33) from 99
L Subtract (-30) from 49
H 18 less than (-77)
E Subtract (-99) from 0
T How much is 0 decreased by (-11)?
S (-42) – (-34) – (-9) - 18

79	-95	132	11	-17	99

B. Mental Math

Give the difference:

1. 53 -25
2. (-6) - 123
3. (-4) - (-9)
4. 6 - 15
5. 16 - (-20)
6. 25 - 43
7. (-30) - (-20)
8. (-19) - 2
9. 30 –(-9)
10. (-19) - (-15)

C. Solve the ff. Problems:

1. Maan deposited P53,400.00 in her account and withdrew P19,650.00 after a week. How much of her money was left in the bank?
2. Two trains start at the same station at the same time. Train A travels 92km/h, while train B travels 82km/h. If the two trains travel in opposite directions, how far apart will they be after an hour?
   If the two trains travel in the same direction, how far apart will they be in two hours?
3. During the Christmas season. The student gov't association was able to solicit 2,356 grocery items and was able to distribute 2,198 to one barangay. If this group decided to distribute 1,201 grocery items to the next barangay, how many more grocery items did they need to solicit?

Summary

In this lesson, you learned how to subtract integers by reversing the process of addition, and by converting subtraction to addition using the negative of the subtrahend.

Lesson 4.3: Fundamental Operations on Integers: Multiplication of Integers
Time: 1 hour

Prerequisite Concepts: Operations on whole numbers, addition and subtraction of integers

About the Lesson: This is the third lesson on operations on integers. The intent of the lesson is to deepen what students have learned in Grade 6, by expounding on the meaning of multiplication of integers.

Objective:

In this lesson; you are expected to:

1. Multiply integers.
2. Apply multiplication of integers in solving problems

Lesson Proper:

I. Activity

Answer the following question.

How do we define multiplication?

We learned that with whole numbers, *multiplication is repeated addition*. For example, 4×3 means three groups of 4. Or, putting it into a real context, 3 cars with 4 passengers each, how many passenger in all? Thus
$4 \times 3 = 4 + 4 + 4 = 12$.

But, if there are 4 cars with 3 passengers each, in counting the total number of passengers, the equation is $3 \times 4 = 3 + 3 + 3 + 3 = 12$. We can say then that $4 \times 3 = 3 \times 4$and

$4 \times 3 = 3 \times 4 = 3 + 3 + 3 + 3 = 12$.

We extend this definition to multiplication of a negative integer by a positive integer. Consider the situation when a boy loses P6 for 3 consecutive days.His total loss for three days is
$-6 \ \times 3$. Hence, we could have
$-6 \ \times 3 = \ -6 \ + \ -6 \ + \ -6 \ = -18$.

II. Questions/Points to Ponder

The following examples illustrate further how integers are multiplied.

Example 1. Multiply : 5 ×(-2)

However,

5 × (-2) = (-2) × (5)

Therefore:

(-2) × (5)= (-2) + (-2) + (-2) + (-2) + (-2) = -10

The result shows that the product of a negative multiplier and a positive multiplicand is a negative integer.

Generalization:Multiplying unlike signs
We know that adding negative numbers means adding their positive equivalents and attaching the negative sign to the result, then

$$a \times (-b) = (-b) \times a = \underbrace{(-b) + (-b) + \cdots + (-b)}_{a\,addends} = -\underbrace{(b + b + \cdots + b)}_{a\,addends} = -ab$$

for any positive integers a and b.

We know that any whole number multiplied by 0 gives 0. Is this true for any integer as well? The answer is YES. In fact, any number multiplied by 0 gives 0. This is known as the ***Zero Property***.

What do we get when we multiply two negative integers?

Example 2. Multiply: (-8) × (-3)

We know that $(-8) \times 3 = -24$.
Therefore,
$-24 + (-8) \times (-3) = (-8) \times 3 + (-8) \times (-3)$
$= (-8) \times [3 + (-3)]$ *(Distributive Law)*
$= (-8) \times 0$ *(3 and -3 are additive inverses)*
$= 0$ *(Zero Property)*

The only number which when added to -24 gives 0 is the additive inverse of -24. Therefore, $(-8) \times (-3)$ is the additive inverse of 24, or $(-8) \times (-3) = 24$

The result shows that the product of two negative integers is a positive integer.

Generalization:Multiplying Two Negative Integers
If a and b are positive integers, then $(-a) \times (-b) = ab$.

Rules in Multiplying Integers:

In multiplying integers, find the product of their positive equivalents.

1. If the integers have the same signs, their product is positive.
2. If the integers have different signs their product is negative.

III. Exercises

A. Find the product of the following:

1. (5)(12)
2. (-8)(4)
3. (-5)(3)(2)
4. (-7)(4)(-2)
5. (3)(8)(-2)
6. (9)(-8)(-9)
7. (-9)(-4)(-6)

MATH DILEMMA

B. How can a person fairly divide 10 apples among 8 children so that each child has the same share.
To solve the dilemma, match the letter in column II with the number that corresponds to the numbers in column I.

Column I	Column II	
1. (6)(-12)	C	270
2. (-13)(-13)	P	-72
3. (19)(-17)	E	300
4. (-15)(29)	K	-323
5. (165)(0)	A	-435
6. (-18)(-15)	M	0
7. (-15)(-20)	L	16
8. (-5)(-5)(-5)	J	-125
9. (-2)(-2)(-2)(-2)	U	169
10. (4)(6)(8)	I	192

____	____	____	____	
5	4	3	7	
____	____	____	____	____
4	1	1	9	7
____	____	____	____	____
8	2	10	6	7

C. Problem Solving
1. Jof has twenty P5 coins in her coin purse. If her niece took 5 of the coins, how much has been taken away?
2. Mark can type 45 words per minute, how many words can Mark type in 30 minutes?
3. Give an arithmetic equation which will solve the following
   a. The messenger came and delivered 6 checks worth PhP50 each. Are you richer or poorer? By how much?
   b. The messenger came and took away 3 checks worth PhP120 each. Are you richer or poorer? By how much?
   c. The messenger came and delivered 12 bills for PhP86 each. Are you richer or poorer? By how much?
   d. The messenger came and took away 15 bills for PhP72 each. Are you richer or poorer? By how much?

Summary

This lesson emphasized the meaning of multiplication to set the rules for multiplying integers. To multiply integers, first find the product of their positive equivalents. If the integers have the same signs, their product is positive. If the integers have different signs their product is negative.

Lesson 4.4: Fundamental Operations on Integers: Division of Integers
Time: 1 hour

Prerequisite Concepts: Addition and subtraction of Integers, Multiplication of Integers

About the Lesson: Like in the previous lessons, this lesson is meant to deepen students' understanding of the division operation on integers. The concept of division used here relies on its relationship to multiplication.

Objective:

In this lesson you are expected to:

1. Find the quotient of two integers.
2. Solve problems involving division of integers.

Lesson Proper:

I. Activity

Answer the following questions:

What is (-51) ÷ (-3)?
What is (-51) ÷ 3?
What is 51 ÷ (-3)?
What are the rules in dividing integers?

II. Questions/Points to Ponder

We have learned that Subtraction is the inverse operation of Addition,
In the same manner, Division is the inverse operation of Multiplication.

Example 1. Find the quotient of (-51) and (-3)

Solution:

Since division is the inverse of multiplication, determine whatnumber multiplied by (-3) produces (-51).

If we ignore the signs for the meantime, we know that

$$3 \times 17 = 51$$

We also know that in order to get a negative product, the factors must have different signs. Hence

$$-3 \ \times 17 = -51$$

Therefore

(-51) ÷ (-3) = 17

Example 2. What is $-57 \ \div 19$?

Solution: $19 \times 3 = 57$

Hence

$$19 \times \ -3 \ = -57$$

Therefore

$$-57 \ \div 19 = -3$$

Example 3. Show why 273 ÷ (–21) = –13.

Solution: (-13) × (-21) = 57

Therefore, 273 ÷ (–21) = –13

Generalization

The quotient of two integers with the same signs is a positive integer and the quotient of two integers having unlike signs is a negative integer.However, division by zero is not possible.

When several operations have to be performed, the GEMDAS rule applies.

Example 4. Perform the indicated operations

1. $2 - 3 \times \quad -4$
2. $4 \times 5 + 72 \div \quad -6$
3. $9 + 6 - \quad -3 \quad \times 12 \div \quad -9$

Solution:

1. $2 - 3 \times \quad -4 \quad = 2 - \quad -12 \quad = 14$
2. $4 \times 5 + 72 \div \quad -6 \quad = 20 + \quad -12 \quad = 8$
3. $9 + 6 - \quad -3 \quad \times 12 \div \quad -9 \quad = 9 + 6 - \quad -36 \quad \div \quad -9 \quad = 9 + 6 - 4 = 11$

III. Exercises:

A. Compute the following
   1. $\quad 10 + 15 \quad - 4 \times 3 + 7 \times (-2)$
   2. $22 \times 9 \div \quad -6 \quad - 5 \times 8$
   3. $36 \div 12 + 53 + (-30)$
   4. $\quad 30 + 26 \quad \div [(-2) \times 7]$
   5. $(124 - 5 \times 12) \div 8$

B. What was the original name for the butterfly?

To find the answer find the quotient of each of the following and write the letter of the problems in the box corresponding to the quotient.

R (-352) ÷

U (-120) ÷ 8

T (128) ÷ -

L (-444) ÷ (-12)

Y (144) ÷ -3

B (108) ÷ 9

E (168) ÷ 6

T (-147) ÷ 7

F (-315) ÷ (-

9	37	-15	-8	-8	28	-16	12	-48

C. Solvethe following problems:
   1. Vergara’s store earned P8750 a week, How much is her average earning in a day?
   2. Russ worked in a factory and earned P7875.00 for 15 days. How much is his earning in a day?
   3. There are 336 oranges in 12 baskets. How many oranges are there in 3 baskets?
   4. A teacher has to divide 280 pieces of graphing paper equally among his 35 students. How many pieces of graphing paper will each student recieve?
   5. A father has 976 sq. meters lot, he has to divide it among his 4 children. What is the share of each child?

D. Complete the three-by-three magic square (that is, the sums of the numbers in each row, in each column and in each of the diagonals are the same) using the numbers -10, -7, -4, -3, 0, 3, 4, 7, 10. What is the sum for each row, column and diagonal?

Summary

Division is the reverse operation of multiplication. Using this definition, it is easy to see that the quotient of two integers with the same signs is a positive integer and the quotient of two integers having unlike signs is a negative integer.

Lesson 5: Properties of the Operations on Integers **Time: 1.5 hours**

Prerequisite Concepts: Addition, Subtraction, Multiplication and Division of Integers

About the Lesson:

This lesson will strengthen the skills of students in performing the fundamental operations of integers. Knowledge of these will serve as an axiom/guide in performing said operations. In addition, this will help students solve problems including real life situations in algebra. This section also discusses how an application of the properties of real numbers in real life situations can be helpful in sustaining harmonious relationships among people.

Objectives

In this lesson, you are expected to:

1. State and illustrate the different properties of the operations on integers
   a. closure
   b. commutative
   c. associative
   d. distributive
   e. identity
   f. inverse
2. Rewrite given expressions according to the given property.

Lesson Proper:

I. A. Activity 1: Try to reflect on these . . .

1. Give at least 5 words synonymous to the word “property”.

Activity 2: PICTIONARY GAME: DRAW AND TELL!

Needed Materials:
5 strips of cartolina with adhesive tape where each of the following words will be written:

- Closure
- Commutative
- Associative
- Distributive
- Identity
- Inverse

Printed Description:

- Stays the same
- Swapping /Interchange
- Bracket Together/Group Together
- Share Out /Spread Out /Disseminate
- One and the Same/Alike
- Opposite/Contrary

Rules of the Game:
The mission of each player holding a strip of cartolina is to let the classmates guess the hidden word by drawing symbols, figures or images on the board without any word.

If the hidden property is discovered, a volunteer from the class will give his/her own meaning of the identified words. Then, from the printed descriptions, he/she can choose the appropriate definition of the disclosed word and verify if his/her initial description is correct.

The game ends when all the words are revealed.

The following questions will be answered as you go along to the next activity.

- What properties of real numbers were shown in the Pictionary Game? Give one example and explain.
- How are said properties seen in real life?

Activity 3: SHOW AND TELL!

Determine what kind of property of real numbers is being illustrated in the following images:

A. Fill in the blanks with the correct numerical values of the motorbike and bicycle riders.

_______ _______

+

equals

+

If ***a*** represents the number of motorbike riders and ***b*** represents the number of bicycle riders, show the mathematical statement for the diagram below.

_______ + _______ = _______ + _______

Guide Questions:

- What operation is used in illustrating the diagram?
- What happened to the terms in both sides of the equation?
- Based on the previous activity, what property is being applied?
- What if the operation is replaced by multiplication, will the same property be applicable? Give an example to prove your answer.
- Define the property.
- Give a real life situation in which the commutative property can be applied.

- Test the property on subtraction and division operations by using simple examples. What did you discover?

B. Fill in the blanks with the correct numerical values of the set of cellphones, ipods and laptops.

________ ________ ________

equals

________ ________ ________

If ***a*** represents the number of cellphones, ***b*** represents the ipods and **c** represents the laptops, show the mathematical statement for the diagram below.

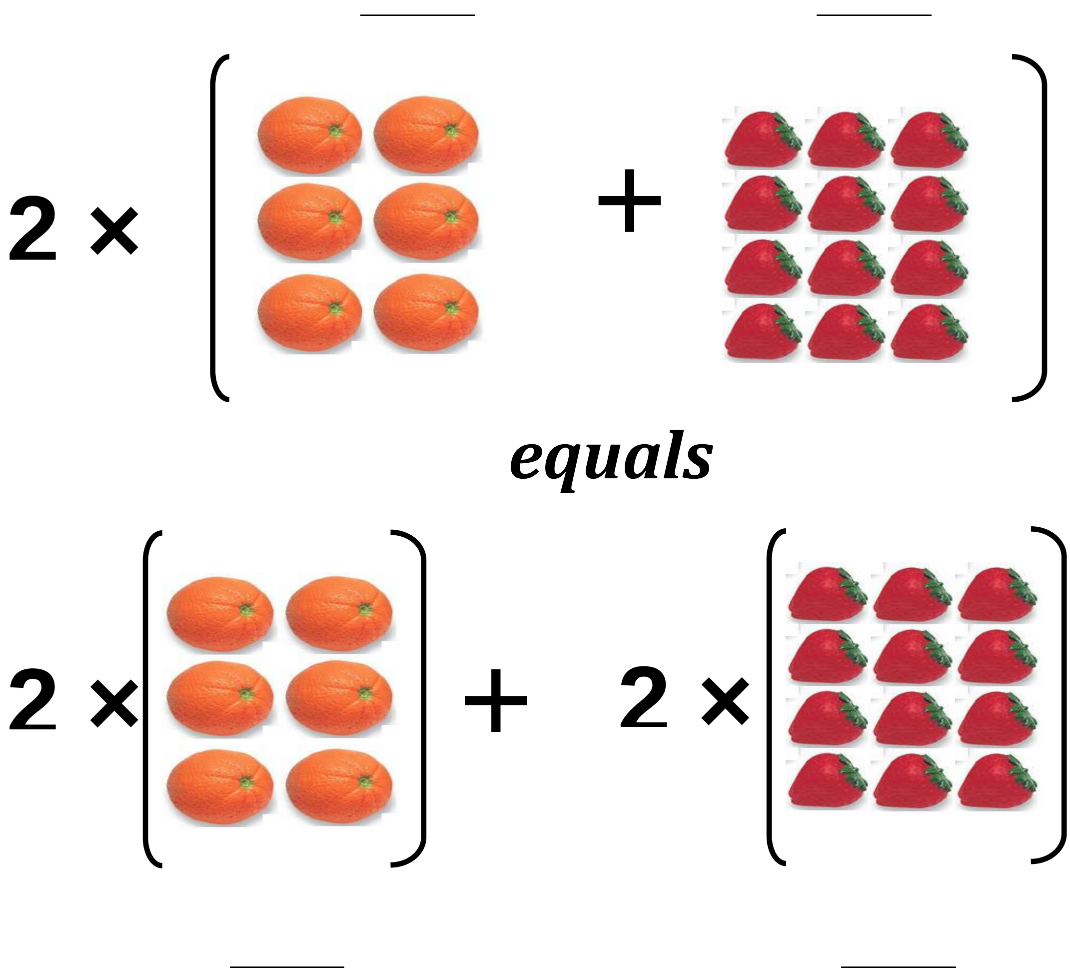
(________ + ________) +________ = ________ + (________ + ________)

Guide Questions:

- What operation is used in illustrating the diagram?
- What happened to the groupings of the given sets that correspond to both sides of the equation?
- Based on the previous activity, what property is being applied?
- What if the operation is replaced by multiplication, will the same property be applicable? Give an example to prove your answer.
- Define the property.

- Give a real life situation wherein associative property can be applied.
- Test the property on subtraction and division operations by using simple examples. What did you discover?

C. Fill in the blanks with the correct numerical values of the set of oranges and set of strawberries.



If ***a*** represents the multiplier in front, ***b*** represents the set of oranges and **c** represents the set of strawberries, show the mathematical statement for the diagram below.

________ (________+________) = _______ • ________ + ________• _______

Guide Questions:

- Based on the previous activity, what property is being applied in the images presented?
- Define the property.
- In the said property can we add/subtract the numbers inside the parentheses and then multiply or perform multiplication first and then addition/subtraction? Give an example to prove your answer.
- Give a real life situation wherein distributive property can be applied.

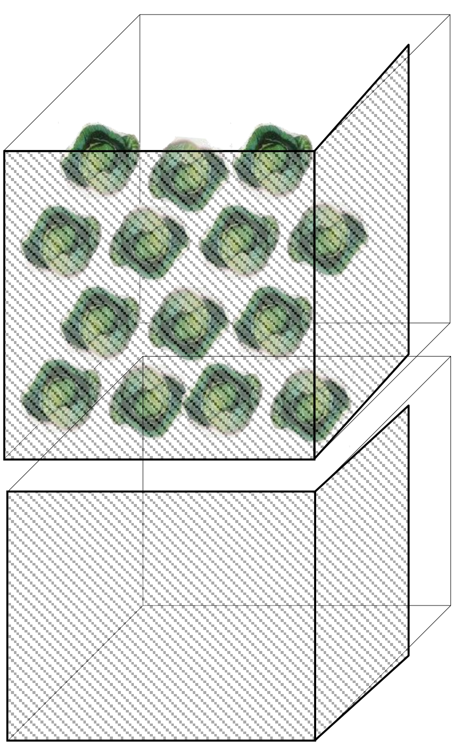
D. Fill in the blanks with the correct numerical representation of the given illustration.

________ ________ ________

Guide Questions:

- Based on the previous activity, what property is being applied in the images presented?
- What will be the result if you add something represented by any number to nothing represented by zero?
- What do you call zero “0” in this case?
- Define the property.
- Is there a number multiplied to any number that will result to that same number? Give examples.
- What property is being illustrated? Define.
- What do you call one “1” in this case?

E. Give the correct mathematical statement of the given illustrations. To do this, refer to the guide questions below.



Guide Questions:

- How many cabbages are there in the crate?
- Using integers, represent “put in 14 cabbages” and “remove 14 cabbages”? What will be the result if you add these representations?
- Based on the previous activity, what property is being applied in the images presented?
- What will be the result if you add something to its negative?
- What do you call the opposite of a number in terms of sign? What is the opposite of a number represented by a?
- Define the property.
- What do you mean by reciprocal and what is the other term used for it?
- What if you multiply a number say 5 by its multiplicative inverse $\frac{1}{5}$, what will be the result?
- What property is being illustrated? Define.

Important Terms to Remember

The following are terms that you must remember from this point on.

1. Closure Property

   Two integers that are added and multiplied remain as integers. The set of integers is closed under addition and multiplication.
2. Commutative Property

   Changing the order of two numbers that are either being added or multiplied does not change the value.
3. Associative Property

   Changing the grouping of numbers that are either being added or multiplied does not change its value.
4. Distributive Property

   When two numbers have been added / subtracted and then multiplied by a factor, the result will be the same when each number is multiplied by the factor and the products are then added / subtracted.
5. Identity Property

   Additive Identity
   - states that the sum of any number and 0 is the given number. Zero, “0” is the additive identity.

   Multiplicative Identity
   - states that the product of any number and 1 is the given number, a • 1 = a. One, “1” is the multiplicative identity.
6. Inverse Property

   In Addition
   - states that the sum of any number and its additive inverse, is zero. The additive inverse of the number a is –a.

   In Multiplication
   - states that the product of any number and its multiplicative inverse or reciprocal, is 1.The multiplicative inverse of the number a is $\frac{1}{a}$.

Notations and Symbols

In this segment, you will learn some of the notations and symbols pertaining to properties of real number applied in the operations of integers.

Closure Property under addition and multiplication	a, b $\in I$, then a+b $\in I$, a•b $\in I$
Commutative property of addition	a + b = b + a
Commutative property of multiplication	ab = ba
Associative property of addition	(a + b) + c = a + (b + c)
Associative property of multiplication	(ab) c = a (bc)
Distributive property	a(b + c) = ab + ac
Additive identity property	a + 0 = a
Multiplicative identity property	a • 1 = a
Multiplicative inverse property	$\frac{1}{a} \cdot a = 1$
Additive inverse property	a + (-a) = 0

III. Exercises

A. Complete the Table: Which property of real number justifies each statement?

Given	Property
1. 0 + (-3) = -3	
2. 2(3 - 5) = 2(3) - 2(5)	
3. (- 6) + (-7) = (-7) + (-6)	
4. 1 x (-9) = -9	
5. $-4 \times -\frac{1}{4} = 1$	
6. 2 x (3 x 7) = (2 x 3) x 7	
7. 10 + (-10) = 0	
8. 2(5) = 5(2)	
9. $1 \times (-\frac{1}{4}) = -\frac{1}{4}$	
10. (-3)(4 + 9) = (-3)(4) + (-3)(9)	

B. Rewrite the following expressions using the given property.

1. 12a – 5a → Distributive Property
2. (7a)b → Associative Property
3. 8 + 5 → Commutative Property
4. -4(1) → Identity Property
5. 25 + (-25) → Inverse Property

C. Fill in the blanks and determine what properties were used to solve the equations.

1. 5 x (_____ + 2) = 0
2. -4 + 4 = ______
3. -6 + 0 = ______
4. (-14 + 14) + 7 = ______
5. 7 x (______ + 7) = 49

Summary

The lesson on the properties or real numbers explains how numbers or values are arranged or related in an equation. It further clarifies that no matter how these numbers are arranged and what processes are used, the composition of the equation and the final answer will still be the same. Our society is much like these equations - composed of different numbers and operations, different people with varied personalities, perspectives and experiences. We can choose to look at the differences and forever highlight one's advantage or superiority over the others. Or we can focus on the commonality among people and altogether, work for the common good. A peaceful society and harmonious relationship starts with recognizing, appreciating and fully maximizing the positive traits that we, as a people, have in common.

Lesson 6: Rational Numbers in the Number Line **Time: 1 hour**

Prerequisite Concepts: Subsets of Real Numbers, Integers

About the lesson:

This lesson is a more in-depth discussion of the set of Rational Numbers and focuses on where they are found in the real number line.

Objective:

In this lesson, you, the students, are expected to

1. Define rational numbers;
2. Illustrate rational numbers on the number line;
3. Arrange rational numbers on the number line.

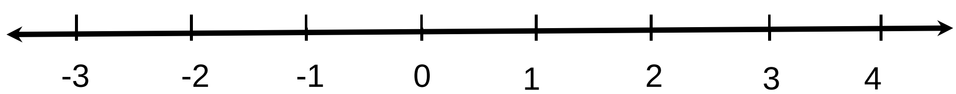
Lesson Proper

I. Activity

Determine whether the following numbers are rational numbers or not.

$$- 2, \pi, \frac{1}{11}, \sqrt[3]{4}, \sqrt{16}, -1.89,$$

Now, try to locate them on the real number line below by plotting:



II. Questions to Ponder

Consider the following examples and answer the questions that follow:

a. 7 ÷ 2 = 3 ½ ,
b. (-25) ÷ 4 = -6 ¼
c. (-6) ÷ (-12) = ½

1. Are quotients integers?
2. What kind of numbers are they?
3. Can you represent them on a number line?

Recall what rational numbers are...

3 ½, -6 ¼, ½, are rational numbers. The word *rational* is derived from the word "ratio" which means quotient. Rational numbers are numbers which can be written as a quotient of two integers, $\frac{a}{b}$ where b ≠ 0.

The following are more examples of rational numbers:

$5 = \frac{5}{1}$ $0.06 = \frac{6}{100}$ $1.3 = \frac{13}{10}$

From the example, we can see that an integer is also a rational number and therefore, integers are a subset of rational numbers. Why is that?

Let's check on your work earlier. Among the numbers given, - 2, π, $\frac{1}{11}$, $\sqrt[3]{4}$, $\sqrt{16}$, -1.89, the numbers π and $\sqrt[3]{4}$ are the only ones that are not rational numbers. Neither can be expressed as a quotient of two integers. However, we can express the remaining ones as a quotient of two intergers:

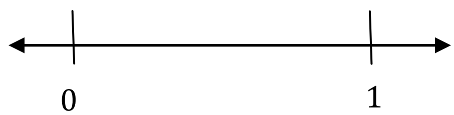
$$-2 = \tfrac{-2}{1},\ \sqrt{16} = 4 = \tfrac{4}{1},\ -1.89 = \frac{-189}{100}$$

Of course, $\frac{1}{11}$ is already a quotient by itself.

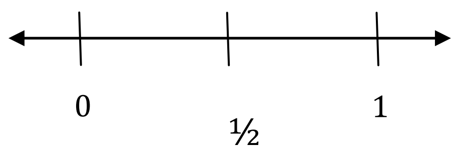
We can locate rational numbers on the real number line.

Example 1. Locate ½ on the number line.

a. Since 0 < ½ < 1, plot 0 and 1 on the number line.

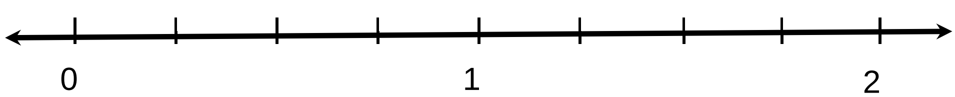


b. Get the midpoint of the segment from 0 to 1. The midpoint now corresponds to ½

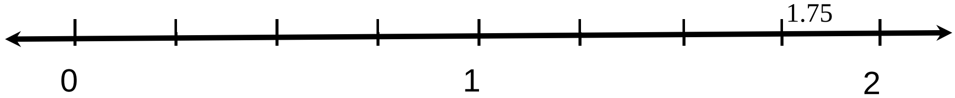


Example 2. Locate 1.75 on the number line.

a. The number 1.75 can be written as $\frac{7}{4}$ and, 1 < $\frac{7}{4}$ < 2. Divide the segment from 0 to 2 into 8 equal parts.

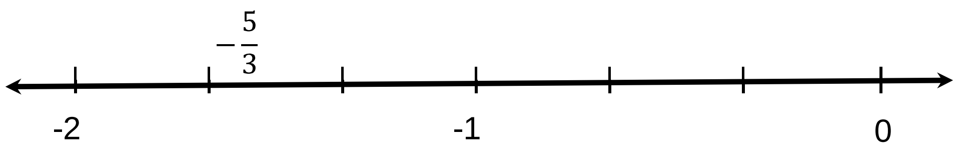


b. The 7th mark from 0 is the point 1.75.



Example 3. Locate the point $-\frac{5}{3}$ on the number line.

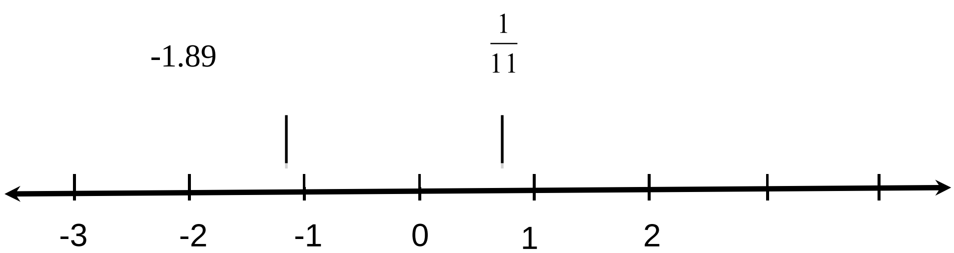
Note that -2 < $-\frac{5}{3}$ < -1. Dividing the segment from -2 to 0 into 6 equal parts, it is easy to plot $-\frac{5}{3}$. The number $-\frac{5}{3}$ is the 5th mark from 0 to the left.



Go back to the opening activity. You were asked to locate the rational numbers and plot them on the real number line. Before doing that, it is useful to arrange them in order from least to greatest. To do this, express all numbers in the same form – either as similar fractions or as decimals. Because integers are easy to locate, they need not take any other form. It is easy to see that

$$-2 < -1.89 < \frac{1}{11} < \sqrt{16}$$

Can you explain why?
Therefore, plotting them by approximating their location gives



III. Exercises

1. Locate and plot the following on a number line (use only one number line).

a. $\frac{-10}{3}$	e. -0.01
b. 2.07	f. $7\frac{1}{9}$
c. $\frac{2}{5}$	g. 0
d. 12	h. $-\frac{1}{6}$

2. Name 10 rational numbers that are greater than -1 but less than 1 and arrange them from least to greatest on the real number line?

3. Name one rational number *x* that satisfies the descriptions below:
   a. $-10 \le x < -9$

   b. $\frac{1}{10} < x < \frac{1}{2}$

c. $3 < x < \pi$

d. $\frac{1}{4} < x < \frac{1}{3}$

e. $-\frac{1}{8} < x < -\frac{1}{9}$

Summary

In this lesson, you learned more about what rational numbers are and where they can be found in the real number line. By changing all rational numbers to equivalent forms, it is easy to arrange them in order, from least to greatest or vice versa.

Lesson 7: Forms of Rational Numbers and Addition and Subtraction of Rational Numbers
Time: 2 hours

Prerequisite Concepts: definition of rational numbers, subsets of real numbers, fractions, decimals

About the Lesson:

Like with any set of numbers, rational numbers can be added and subtracted. In this lesson, you will learn techniques in adding and subtracting rational numbers. Techniques include changing rational numbers into various forms convenient for the operation as well as estimation and computation techniques.

Objectives:

In this lesson, you are expected to:

1. Express rational numbers from fraction form to decimal form (terminating and repeating and non-terminating) and vice versa;
2. Add and subtract rational numbers;
3. Solve problems involving addition and subtraction of rational numbers.

Lesson Proper:

A. Forms of Rational Numbers

I. Activity

1. Change the following rational numbers in fraction form or mixed number form to decimal form:

a. $-\frac{1}{4}$ = ______ d. $\frac{5}{2}$ = ______

b. $\frac{3}{10}$ = ______ e. $-\frac{17}{10}$ = ______

c. $3\frac{5}{100}$ = _____ f. $-2\frac{1}{5}$ = ______

2. Change the following rational numbers in decimal form to fraction form.

a. 1.8 = ______ d. -0.001 = ______

b. - 3.5 = ______ e. 10.999 = ______

c. -2.2 = ______ f. $0.\overline{11}$ = ______

II. Discussion

Non-decimal Fractions

There is no doubt that most of the above exercises were easy for you. This is because all except item 2f are what we call decimal fractions. These numbers are all parts of powers of 10. For example, $-\frac{1}{4}$ = $\frac{25}{100}$ which is easily convertible to a decimal form, 0.25. Likewise, the number -3.5 = $-3\frac{5}{10}=-\frac{35}{10}$.

What do you do when the rational number is not a decimal fraction? How do you convert from one form to the other?

Remember that a rational number is a quotient of 2 integers. To change a rational number in fraction form, you need only to divide the numerator by the denominator.

Consider the number $\frac{1}{8}$. The smallest power of 10 that is divisible by 8 is 1000. But, $\frac{1}{8}$ means you are dividing 1 whole unit into 8 equal parts. Therefore, divide 1 whole unit first into 1000 equal parts and then take $\frac{1}{8}$ of the thousandths part. That is equal to $\frac{125}{1000}$ or 0.125.

Example: Change $\frac{1}{16}$, $\frac{9}{11}$ and $-\frac{1}{3}$ to their decimal forms.

The smallest power of 10 that is divisible by 16 is 10,000. Divide 1 whole unit into 10,000 equal parts and take $\frac{1}{16}$ of the ten thousandths part. That is equal to $\frac{625}{10000}$ or 0.625. You can obtain the same value if you perform the long division $1 \div 16$.

Do the same for $\frac{9}{11}$. Perform the long division $9 \div 11$ and you should obtain $0.\overline{81}$. Therefore, $\frac{9}{11}$ = $0.\overline{81}$. Also, $-\frac{1}{3}=-0.\bar{3}$. Note that both $\frac{9}{11}$ and $-\frac{1}{3}$ are non-terminating but repeating decimals.

To change rational numbers in decimal forms, express the decimal part of the numbers as a fractional part of a power of 10. For example, -2.713 can be changed initially to $-2\frac{713}{1000}$ and then changed to $-\frac{2173}{1000}$.

What about non-terminating but repeating decimal forms? How can they be changed to fraction form? Study the following examples:

Example 1: Change $0.\overline{2}$ to its fraction form.
Solution: Let

$$r = 0.222...$$
$$10r = 2.222...$$

Since there is only 1 repeated digit, multiply the first equation by 10.

Then subtract the first equation from the second equation and obtain

$$9r = 2.0$$
$$r = \frac{2}{9}$$

Therefore, $0.\overline{2} = \frac{2}{9}$.

Example 2. Change $-1.\overline{35}$ to its fraction form.

Solution: Let

$$r = -1.353535...$$
$$100r = -135.353535...$$

Since there are 2 repeated digits, multiply the first equation by 100. In general, if there are n repeated digits, multiply the first equation by 10^n.

Then subtract the first equation from the second equation and obtain

$$99r = -134$$
$$r = -\frac{134}{99} = -1\frac{35}{99}$$

Therefore, $-1.\overline{35} = -\frac{135}{99}$.

B. Addition and Subtraction of Rational Numbers in Fraction Form

I. Activity

Recall that we added and subtracted whole numbers by using the number line or by using objects in a set.

Using linear or area models, find the sum or difference.

a. $\frac{3}{5} + \frac{1}{5} =$ ______

b. $\frac{1}{8} + \frac{5}{8} =$ ______

c. $\frac{10}{11} - \frac{3}{11} =$ ______

d. $3\frac{6}{7} - 1\frac{2}{7} =$ ______

Without using models, how would you get the sum or difference?

Consider the following examples:

1. $\frac{1}{6} + \frac{1}{2} = \frac{1}{6} + \frac{3}{6} = \frac{4}{6}$ $0r$ $\frac{2}{3}$
2. $\frac{6}{7} + -\frac{2}{3} = \frac{18}{21} + -\frac{14}{21} = \frac{4}{21}$

3. $-\frac{4}{3} + -\frac{1}{5} = -\frac{20}{15} + -\frac{3}{15} = -\frac{23}{15}$ $or - 1\frac{8}{15}$
4. $\frac{14}{5} - \frac{4}{7} = \frac{98}{35} - \frac{20}{35} = \frac{78}{35}$ or $2\frac{8}{35}$
5. $-\frac{7}{12} - -\frac{2}{3} = -\frac{7}{12} - -\frac{8}{12} = \frac{-7+8}{12} = \frac{1}{12}$
6. $-\frac{1}{6} - -\frac{11}{20} = -\frac{10}{60} - -\frac{33}{60} = \frac{-10+33}{60} = \frac{23}{60}$

Answer the following questions:
1. Is the common denominator always the same as one of the denominators of the given fractions?
2. Is the common denominator always the greater of the two denominators?
3. What is the least common denominator of the fractions in each example?
4. Is the resulting sum or difference the same when a pair of dissimilar fractions is replaced by any pair of similar fractions?

Problem: Copy and complete the fraction magic square. The sum in each row, column, and diagonal must be 2.

a ____	$\frac{1}{2}$	b ____
$\frac{7}{5}$	$\frac{1}{3}$	c ____
d ____	e ____	$\frac{2}{5}$

» What are the values of a, b, c, d and e?

Important things to remember

To Add or Subtract Fraction

- With the same denominator,
  If a, b and c denote integers, and b ≠ 0, then
  $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$ and $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$

- With different denominators, $\frac{a}{b}$ and $\frac{c}{d}$, where b ≠ 0 and d ≠ 0
  If the fractions to be added or subtracted are dissimilar

  » Rename the fractions to make them similar whose denominator is the least common multiple of *b* and *d*.
  » Add or subtract the numerators of the resulting fractions.
  » Write the result as a fraction whose numerator is the sum or difference of the numerators and whose denominator is the least common multiple of *b* and *d*.

<u>Examples</u>:

To Add:

a. $\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{2}{7}$

To Subtract:

a. $\frac{5}{7} - \frac{2}{7} = \frac{5-2}{7} = \frac{3}{7}$

b. $\frac{2}{5} + \frac{1}{4}$

LCM/LCD of 5 and 4 is 20

$\frac{2}{5} + \frac{1}{4} = \frac{8}{20} + \frac{5}{20} = \frac{8+5}{20} = \frac{13}{20}$

b. $\frac{4}{5} - \frac{1}{4}$

$\frac{4}{5} - \frac{1}{4} = \frac{16}{20} - \frac{5}{20} = \frac{16-5}{20} = \frac{11}{20}$

II. Question to Ponder (Post –Activity Discussion)

Let us answer the questions posed in activity.
You were asked to find the sum or difference of the given fractions.

a. $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

b. $\frac{1}{8} + \frac{5}{8} = \frac{6}{8}$ *or* $\frac{3}{4}$

c. $\frac{10}{11} - \frac{3}{11} = \frac{7}{11}$

d. $3\frac{6}{7} - 1\frac{2}{7} = 2\frac{4}{7}$

Without using the models, how would you get the sum or difference?
You would have to apply the rule for adding or subtracting similar fractions.

1. Is the common denominator always the same as one of the denominators of the given fractions?
   Not always. Consider $\frac{2}{5} + \frac{3}{4}$*. Their least common denominator is 20 not 5 or 4.*

2. Is the common denominator always the greater of the two denominators?
   Not always. The least common denominator is always greater than or equal to one of the two denominators and it may not be the greater of the two denominators.

3. What is the least common denominator of the fractions in each example?
   (1) *6* (2) *21* (3) *15* (4) *35* (5) *12* (6) *60*

4. Is the resulting sum or difference the same as when a pair of dissimilar fractions is replaced by any pair of similar fractions?
   Yes, for as long as the replacement fractions are equivalent to the original fractions.

III. Exercises

Do the following exercises.

a. Perform the indicated operations and express your answer in simplest form.

1. $\frac{2}{9} + \frac{3}{9} + \frac{1}{9}$
2. $\frac{6}{5} + \frac{3}{5} + \frac{4}{5}$
3. $\frac{2}{5} + \frac{7}{10}$
4. $\frac{16}{24} - \frac{6}{12}$
5. $2\frac{5}{12} - \frac{2}{3}$
9. $\frac{7}{9} - \frac{1}{12}$
10. $11\frac{5}{9} - 7\frac{5}{6}$
11. $\frac{1}{4} + \frac{2}{3} - \frac{1}{2}$
12. $10 - 3\frac{5}{11}$
13. $\frac{7}{20} + \frac{3}{8} + \frac{2}{5}$

6. $8\frac{1}{4} + \frac{2}{7}$
7. $3\frac{1}{4} + 6\frac{2}{3}$
8. $9\frac{5}{7} - 3\frac{2}{7}$
14. $\frac{5}{12} + \frac{4}{9} - \frac{1}{4}$
15. $2\frac{5}{8} + \frac{1}{2} + 7\frac{3}{4}$

b. Give the number asked for.
1. What is three more than three and one-fourth?
2. Subtract from $15\frac{1}{2}$ the sum of $2\frac{1}{3}$ *and* $4\frac{2}{5}$. What is the result?
3. Increase the sum of $6\frac{3}{14}$ *and* $2\frac{2}{7}$ *by* $3\frac{1}{2}$. What is the result?
4. Decrease $21\frac{3}{8}$ *by* $5\frac{1}{5}$. What is the result?
5. What is $-8\frac{4}{5}$ *minus* $3\frac{2}{7}$?

c. Solve each problem.
1. Michelle and Corazon are comparing their heights. If Michelle's height is $120\frac{3}{4}$ cm. and Corazon's height is $96\frac{1}{3}$ cm. What is the difference in their heights?
2. Angel bought $6\frac{3}{4}$ meters of silk, $3\frac{1}{2}$ meters of satin and $8\frac{2}{5}$ meters of velvet. How many meters of cloth did she buy?
3. Arah needs $10\frac{1}{4}$ kg. of meat to serve 55 guests, If she has $3\frac{1}{2}$ kg of chicken, a $2\frac{3}{4}$ kg of pork, and $4\frac{1}{4}$ kg of beef, is there enough meat for 55 guests?
4. Mr. Tan has $13\frac{2}{5}$ liters of gasoline in his car. He wants to travel far so he added $16\frac{1}{2}$ liters more. How many liters of gasoline is in the tank?
5. After boiling, the $17\frac{3}{4}$ liters of water was reduced to $9\frac{2}{3}$ liters. How much water has evaporated?

C. Addition and Subtraction of Rational Numbers in Decimal Form

There are 2 ways of adding or subtracting decimals.

1. Express the decimal numbers in fractions then add or subtract as described earlier.

Example:

Add: 2.3 + 7.21

$2\frac{3}{10} + 7\frac{21}{100}$

$2\frac{30}{100} + 7\frac{21}{100}$

$(2 + 7) + \frac{30+21}{100}$

$9 + \frac{51}{100} = 9\frac{51}{100}$ or 9.51

Subtract:: 9.6 – 3.25

$9\frac{6}{10} - 3\frac{25}{100}$

$9\frac{60}{100} - 3\frac{25}{100}$

$(9 - 3) + \frac{60-25}{100}$

$6 + \frac{35}{100} = 6\frac{35}{100}$ or 6.35

2. Arrange the decimal numbers in a column such that the decimal points are aligned, then add or subtract as with whole numbers.
   Example:

Add: 2.3 + 7.21 Subtract: 9.6- 3.25

$$\begin{array}{r} 2.3 \\ +\ \underline{\ \ 7.21} \\ 9.51 \end{array} \qquad \begin{array}{r} 9.6 \\ \underline{-\ 3.25} \\ 6.35 \end{array}$$

Exercises:
1. Perform the indicated operation.

1) 1,902 + 21.36 + 8.7
2) 45.08 + 9.2 + 30.545
3) 900 + 676.34 + 78.003
4) 0.77 + 0.9768 + 0.05301
5) 5.44 – 4.97
6) 700 – 678.891
7) 7.3 – 5.182
8) 51.005 – 21.4591
9) (2.45 + 7.89) – 4.56
10) (10 – 5.891) + 7.99

2. Solve the following problems:

a. Helen had P7500 for shopping money. When she got home, she had P132.75 in her pocket. How much did she spend for shopping?
b. Ken contributed P69.25, while John and Hanna gave P56.25 each for their gift to Teacher Daisy. How much were they able to gather altogether?
c. Ryan said, "I'm thinking of a number N. If I subtract 10.34 from N, the difference is 1.34." What was Ryan's number?
d. Agnes said, "I'm thinking of a number N. If I increase my number by 56.2, the sum is 14.62."What was Agnes number?
e. Kim ran the 100-meter race in 135.46 seconds. Tyron ran faster by 15.7 seconds. What was Tyron's time for the 100-meter dash?

SUMMARY

This lesson began with some activities and instruction on how to change rational numbers from one form to another and proceeded to discuss addition and subtraction of rational numbers. The exercises given were not purely computational. There were thought questions and problem solving activities that helped in deepening one's understanding of rational numbers.

Lesson 8: Multiplication and Division of Rational Numbers
Time: 2 hours

Prerequisite Concepts: addition and subtraction of rational numbers, expressing rational numbers in different forms

About the lesson:

In this lesson, you will learn how to multiply and divide rational numbers. While there are rules and algorithms to remember, this lesson also shows why those rules and algorithms work.

Objectives:

In this lesson, you are expected to:

1. Multiply rational numbers;
2. Divide rational numbers;
3. Solve problems involving multiplication and division of rational numbers.

Lesson Proper

A. Models for the Multiplication and Division

I. Activity:

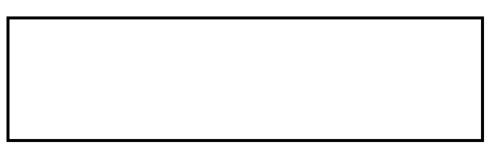
Make a model or a drawing to show the following:

1. A pizza is divided into 10 equal slices. Kim ate $\frac{3}{5}$ of $\frac{1}{2}$ of the pizza. What part of the whole pizza did Kim eat?
2. Miriam made 8 chicken sandwiches for some street children. She cut up each sandwich into 4 triangular pieces. If a child can only take a piece, how many children can she feed?

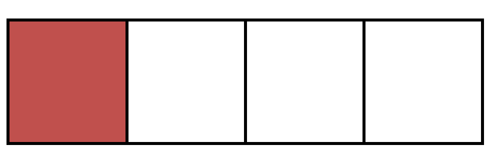
Can you make a model or a drawing to help you solve these problems?

A model that we can use to illustrate multiplication and division of rational numbers is the <u>area model</u>.

What is $\frac{1}{4} \times \frac{1}{3}$? Suppose we have one bar of chocolate represent 1 unit.

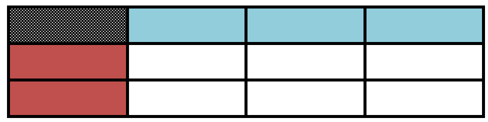


Divide the bar first into 4 equal parts vertically. One part of it is $\frac{1}{4}$



Then, divide each fourth into 3 equal parts, this time horizontally to make the divisions easy to see. One part of the horizontal division is $\frac{1}{3}$.

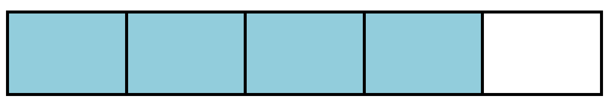
$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$



There will be 12 equal-sized pieces and one piece is $\frac{1}{12}$. But, that one piece is $\frac{1}{3}$ of $\frac{1}{4}$, which we know from elementary mathematics to mean $\frac{1}{3} \times \frac{1}{4}$.

What about a model for division of rational numbers?

Take the division problem: $\frac{4}{5} \div \frac{1}{2}$. One unit is divided into 5 equal parts and 4 of them are shaded.



Each of the 4 parts now will be cut up in halves

Since there are 2 divisions per part (i.e. $\frac{1}{5}$) and there are 4 of them (i.e. $\frac{4}{5}$), then there will be 8 pieces out of 5 original pieces or $\frac{4}{5} \div \frac{1}{2} = \frac{8}{5}$.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions posed in the opening activity.

1. A pizza is divided into 10 equal slices. Kim ate $\frac{3}{5}$ of $\frac{1}{2}$ of the pizza. What part of the whole pizza did Kim eat?

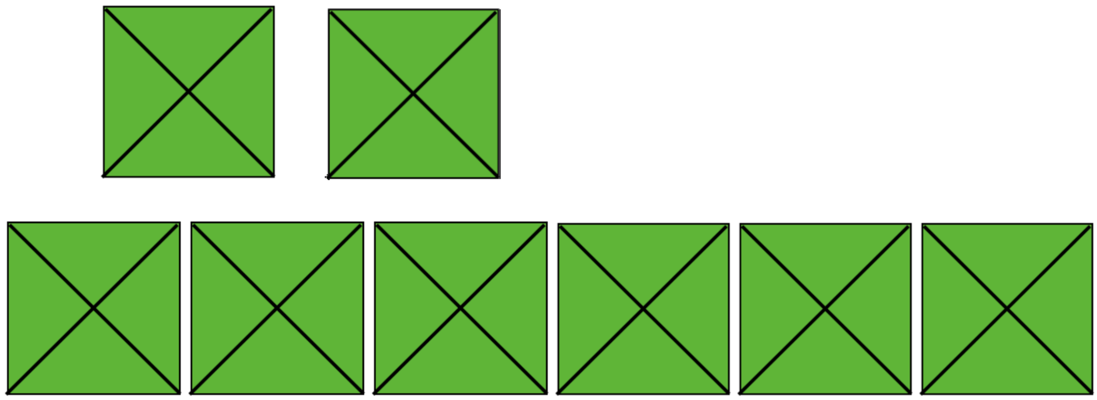
½

3/5

$$\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$$

Kim ate $\frac{3}{10}$ of the whole pizza.

2. Miriam made 8 chicken sandwiches for some street children. She cut up each sandwich into 4 triangular pieces. If a child can only take a piece, how many children can she feed?



The equation is $8 \div \frac{1}{4} = 32$. Since there are 4 fourths in one sandwich, there will be 4 x 8 = 32 triangular pieces and hence, 32 children will be fed.
How then can you multiply or divide rational numbers without using models or drawings?

Important Rules to Remember
The following are rules that you must remember. From here on, the symbols to be used for multiplication are any of the following: ●, x, $\times$, or *x.*

1. To multiply rational numbers in fraction form simply multiply the numerators and multiply the denominators.

   In symbol, $\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$ where b and d are NOT equal to zero, (b ≠ 0; d ≠ 0)

2. To divide rational numbers in fraction form, you take the reciprocal of the second fraction (called the divisor) and multiply it by the first fraction.

   In symbol, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$ where b, c, and d are NOT equal to zero.

<u>Example:</u>
Multiply the following and write your answer in simplest form

a. $\frac{3}{7} \bullet \frac{2}{5}$
$\frac{3}{7} \bullet \frac{2}{5} = \frac{3 \: x \: 2}{7 \: x \: 5} = \frac{6}{35}$

b. $4\frac{1}{3} \bullet 2\frac{1}{4}$
$\frac{13}{3} \bullet \frac{9}{4} = \frac{13 \bullet 3 \bullet 3}{3 \bullet 4} = \frac{13 \bullet 3}{4} = \frac{39}{4} \; or \; 9\frac{3}{4}$

The easiest way to solve for this number is to change mixed numbers to an improper fraction and then multiply it. Or use prime factors or the greatest common factor, as part of the multiplication process.

Divide: $\frac{8}{11} \div \frac{2}{3}$

$$\frac{8}{11} \div \frac{2}{3} = \frac{8}{11} \bullet \frac{3}{2}$$
$$= \frac{2\bullet 4}{11} \bullet \frac{3}{2}$$
$$\frac{4\bullet 3}{11} = \frac{12}{11} \text{ or } 1\frac{1}{11}$$

Take the reciprocal of $\frac{2}{3}$, which is $\frac{3}{2}$ then multiply it with the first fraction. Using prime factors, it is easy to see that 2 can be factored out of the numerator then cancelled out with the denominator, leaving 4 and 3 as the remaining factors in the numerator and 11 as the remaining factors in the denominator.

III. Exercises.

Do the following exercises. Write your answer on the spaces provided:

1. Find the products:

a. $\frac{5}{6} \bullet \frac{2}{3}$

b. $7 \bullet \frac{2}{3}$

c. $\frac{4}{20} \bullet \frac{2}{5}$

d. $10\frac{5}{6} \bullet 3\frac{1}{3}$

e. $-\frac{9}{20} \bullet \frac{25}{27}$

f. $4\frac{1}{2} \bullet 5\frac{2}{3}$

g. $\frac{2}{15} \bullet \frac{3}{4}$

h. $\frac{1}{6} \bullet \frac{2}{3} \bullet \frac{1}{4}$

i. $-\frac{5}{6} \bullet \frac{2}{3} \bullet -\frac{12}{15}$

j. $\frac{9}{16} \bullet \frac{4}{15} \bullet -2$

B. Divide:

1. $20 \div \frac{2}{3}$
2. $\frac{5}{12} \div -\frac{3}{4}$
3. $\frac{5}{50} \div \frac{20}{35}$
4. $5\frac{3}{4} \div 6\frac{2}{3}$
5. $\frac{9}{16} \div \frac{3}{4} \div \frac{1}{6}$
6. $\frac{8}{15} \div \frac{12}{25}$
7. $13\frac{1}{6} \div -2$
8. $-\frac{5}{6} \div -\frac{10}{14}$
9. $-\frac{2}{9} \div \frac{11}{15}$
10. $\frac{15}{6} \div \frac{2}{3} \div \frac{5}{8}$

C. Solve the following:

1. Julie spent $3\frac{1}{2}$ hours doing her assignment. Ken did his assignment for $1\frac{2}{3}$ times as many hours as Julie did. How many hours did Ken spend doing his assignment?
2. How many thirds are there in six-fifths?
3. Hanna donated $\frac{2}{5}$ of her monthly allowance to the Iligan survivors. If her monthly allowance is P3500, how much did she donate?
4. The enrolment for this school year is 2340. If $\frac{1}{6}$ are sophomores and $\frac{1}{4}$ are seniors, how many are freshmen and juniors?
5. At the end of the day, a store had 2/5 of a cake leftover. The four employees each took home the same amount of leftover cake. How much did each employee take home?

B. Multiplication and Division of Rational Numbers in Decimal Form

This unit will draw upon your previous knowledge of multiplication and division of whole numbers. Recall the strategies that you learned and developed when working with whole numbers.

Activity:

1. Give students several examples of multiplication sentences with the answers given. Place the decimal point in an incorrect spot and ask students to explain why the decimal place does not go there and explain where it should go and why.

   Example:

   215.2 x 3.2 = 68.864

2. Five students ordered buko pie and the total cost was P135.75. How much did each student have to pay if they shared the cost equally?

Questions and Points to Ponder:

1. In multiplying rational numbers in decimal form, note the importance of knowing where to place the decimal point in a product of two decimal numbers. Do you notice a pattern?
2. In dividing rational numbers in decimal form, how do you determine where to place the decimal point in the quotient?

Rules in Multiplying Rational Numbers in Decimal Form

1. Arrange the numbers in a vertical column.
2. Multiply the numbers, as if you are multiplying whole numbers.
3. Starting from the rightmost end of the product, move the decimal point to the left the same number of places as the sum of the decimal places in the multiplicand and the multiplier.

Rules in Dividing Rational Numbers in Decimal Form

1. If the divisor is a whole number, divide the dividend by the divisor applying the rules of a whole number. The position of the decimal point is the same as that in the dividend.
2. If the divisor is not a whole number, make the divisor a whole number by moving the decimal point in the divisor to the rightmost end, making the number seem like a whole number.
3. Move the decimal point in the dividend to the right the same number of places as the decimal point was moved to make the divisor a whole number.
4. Lastly divide the new dividend by the new divisor.

Exercises:

A. Perform the indicated operation

1. 3.5 ÷ 2
2. 78 x 0.4
3. 9.6 x 13
6. 27.3 x 2.5
7. 9.7 x 4.1
8. 3.415 ÷ 2.5

4. 3.24 ÷ 0.5
5. 1.248 ÷ 0.024
9. 53.61 x 1.02
10. 1948.324 ÷ 5.96

B. Finds the numbers that when multiplied give the products shown.

1.
$$\begin{array}{r} \square.\square \\ \times \quad \square \\ \hline 10.6 \end{array}$$

2.
$$\begin{array}{r} \square.\square \\ \times \quad \square \\ \hline 16.8 \end{array}$$

3.
$$\begin{array}{r} \square.\square \\ \times \quad \square \\ \hline 21.6 \end{array}$$

4.
$$\begin{array}{r} \square.\square \\ \times \quad \square \\ \hline 9.5 \end{array}$$

5.
$$\begin{array}{r} \square.\square\,\square \\ \times \quad \square \\ \hline 21.98 \end{array}$$

Summary

In this lesson, you learned to use the area model to illustrate multiplication and division of rational numbers. You also learned the rules for multiplying and dividing rational numbers in both the fraction and decimal forms. You solved problems involving multiplication and division of rational numbers.

Lesson 9: Properties of the Operations on Rational Numbers
Time: 1.5 hours

Pre-requisite Concepts: Operations on rational numbers

About the Lesson: The purpose of this lesson is to use properties of operations on rational numbers when adding, subtracting, multiplying and dividing rational numbers.

Objectives:

In this lesson, you are expected to

1. Describe and illustrate the different properties of the operations on rational numbers.
2. Apply the properties in performing operations on rational numbers.

Lesson Proper:

I. Activity

Pick a Pair

$\frac{2}{14}$	$\frac{3}{5}$	0	1	$\frac{13}{40}$
	$\frac{13}{12}$	$\frac{1}{3}$	$\frac{3}{20}$	

From the box above, pick the correct rational number to be placed in the spaces provided to make the equation true.

1. $\frac{3}{14}$ + ____ = $\frac{5}{14}$
2. ____ + $\frac{3}{14}$ = $\frac{5}{14}$
3. $\frac{1}{3}\ x$ ___ = 0
4. 1 x ____ = $\frac{3}{5}$
5. $\frac{2}{3}$ + ____ = $\frac{2}{3}$

6. $\frac{1}{2} + \frac{1}{4} + \frac{1}{3}$ = ____
7. $\frac{1}{2} + \frac{1}{4}$ + ___ = $\frac{13}{12}$
8. $\frac{2}{5}\times\left(\_\_\_\times\frac{3}{4}\right)=\frac{3}{20}$
9. $\frac{2}{5}\ x\ \frac{1}{2}\ x\ \frac{3}{4}$ = ____
10. $\frac{1}{2}\ x\ \frac{2}{5} + \frac{1}{4}$ = $\frac{1}{2}\ x\ \frac{2}{5}$ +
$\frac{1}{2}\ x\ \frac{1}{4}$ = ____

Answer the following questions:

1. What is the missing number in item 1?
2. How do you compare the answers in items 1 and 2
3. What about item 3? What is the missing number?
4. In item 4, what number did you multiply with 1 to get $\frac{3}{5}$?

5. What number should be added to $\frac{2}{3}$ in item 5 to get the same number?
6. What is the missing number in items 6 and 7?
7. What can you say about the grouping in items 6 and 7?
8. What do you think are the answers in items 8 and 9?
9. What operation did you apply in item 10?

Problem:

Consider the given expressions:

a. $\frac{1}{4} + \frac{1}{8} + \frac{1}{2} + \frac{2}{3} = \frac{1}{4} + \frac{1}{2} + \frac{2}{3} + \frac{1}{8}$

b. $\frac{2}{15} \bullet \frac{5}{6} = \frac{5}{6} \bullet \frac{2}{15}$

* Are the two expressions equal? If yes, state the property illustrated.

PROPERTIES OF RATIONAL NUMBERS (ADDITION & MULTIPLICATION)

1. CLOSURE PROPERTY: For any two rational numbers. $\frac{a}{b}$ and $\frac{c}{d}$, their sum $\frac{a}{b} + \frac{c}{d}$ and product $\frac{a}{b}$ x $\frac{c}{d}$ is also rational.

   For example:

   a. $\frac{3}{4} + \frac{2}{4} = \frac{3+2}{4} = \frac{5}{4}$

   b. $\frac{3}{4} \bullet \frac{2}{4} = \frac{6}{16}$ or $\frac{3}{8}$

2. COMMUTATIVE PROPERTY: For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$,

   i. $\frac{a}{b} + \frac{c}{d} = \frac{b}{d} + \frac{a}{b}$

   ii. $\frac{a}{b} \bullet \frac{c}{d} = \frac{c}{d}$ and $\frac{a}{b}$

   where a, b, c and d are integers and b and d are not equal to zero.

   For example:

   a. $\frac{2}{7} + \frac{1}{3} = \frac{1}{3} + \frac{1}{7}$

   b. $\frac{6}{7} \bullet \frac{2}{3} = \frac{3}{3} \bullet \frac{6}{7}$

3. ASSOCIATIVE PROPERTY: For any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$

   i. $\frac{a}{b} + \frac{c}{d} + \frac{e}{f} = \frac{a}{b} + \frac{c}{d} + \frac{e}{f}$

   ii. $\frac{a}{b} \bullet \frac{c}{d} \bullet \frac{e}{f} = \frac{a}{b} \bullet \frac{c}{d} \bullet \frac{e}{f}$

where a, b, c, d, e and f are integers and b, d and f are not equal to zero.
For example:

a. $\frac{3}{5} + \frac{2}{3} + \frac{1}{4} = \frac{3}{5} + \frac{2}{3} + \frac{1}{4}$

b. $\frac{1}{4} \bullet \frac{3}{4} \bullet \frac{2}{3} = \frac{1}{4} \bullet \frac{3}{4} \bullet \frac{2}{3}$

4. DISTRIBUTIVE PROPERTY of multiplication over addition for rational numbers.
   If $\frac{a}{b}, \frac{c}{d}, and\ \frac{e}{f}$ are any rational numbers, then $\frac{a}{b} \bullet \frac{c}{d} + \frac{e}{f} = \frac{a}{b} \bullet \frac{c}{d} + \frac{a}{b} \bullet \frac{e}{f}$

   For example: $\frac{3}{7} \bullet \frac{2}{3} + \frac{7}{8} = \frac{3}{7} \bullet \frac{2}{3} + \frac{3}{7} \bullet \frac{7}{8}$

5. DISTRIBUTIVE PROPERTY of multiplication over subtraction for rational numbers.
   If $\frac{a}{b}, \frac{c}{d}, and\ \frac{e}{f}$ are any rational numbers, then $\frac{a}{b} \bullet \frac{c}{d} - \frac{e}{f} = \frac{a}{b} \bullet \frac{c}{d} - \frac{a}{b} \bullet \frac{e}{f}$

   For example: $\frac{3}{10} \bullet \frac{2}{3} - \frac{2}{8} = \frac{3}{10} \bullet \frac{2}{3} - \frac{3}{10} \bullet \frac{2}{8}$

6. IDENTITY PROPERTY
   Addition: Adding 0 to a number will not change the identity or value of that number.

   $\frac{a}{b} + 0 = \frac{a}{b}$

   For example: $\frac{1}{2} + 0 = \frac{1}{2}$

   Multiplication: Multiplying a number by 1 will not change the identity or value of that number.

   $\frac{a}{b} \bullet 1 = \frac{a}{b}$

   For example: $\frac{3}{5} \bullet 1 = \frac{3}{5}$

7. ZERO PROPERTY OF MULTIPLICATION: Any number multiplied by zero equals 0, i. e. $\frac{a}{b} \bullet 0 = 0$
   For example: $\frac{2}{7} \bullet 0 = 0$

II. Question to Ponder (Post-Activity Discussion)

Let us answer the questions posed in the opening activity.

1. What is the missing number in item1? » $\frac{2}{14}$
2. How do you compare the answers in items 1 and 2? » *The answer is the same, the order of the numbers is not important.*
3. What about item 3? What is the missing number? » *The missing number is 0. When you multiply a number with zero the product is zero.*
4. In item 4, what number did you multiply with 1 to get $\frac{3}{5}$? » $\frac{3}{5}$, *When you multiply a number by one the answer is the same.*
5. What number should be added to $\frac{2}{3}$ in item 5 to get the same number? » 0, *When you add zero to any number, the value of the number does not change.*
6. What do you think is the missing number in items 6 and 7?» $\frac{13}{12}$
7. What can you say about the grouping in items 6 and 7? » *The groupings are different but they do not affect the sum.*
8. What do you think are the answers in items 8 and 9? » *The answer is the same in both items,* $\frac{3}{20}$.
9. What operation did you apply in item 10? » *The Distributive Property of Multiplication over Addition*

III. Exercises:

Do the following exercises. Write your answer in the spaces provided.

A. State the property that justifies each of the following statements.

1. $\frac{2}{3} + \frac{5}{8} = \frac{5}{8} + \frac{2}{3}$ ________________________________

2. $1 \bullet \frac{9}{35} = \frac{9}{35}$ ________________________________

3. $\frac{4}{5} \bullet \frac{3}{4} + \frac{2}{3} = \frac{4}{5} \bullet \frac{3}{4} + \frac{4}{5} \bullet \frac{2}{3}$ ________________________________

4. $\frac{3}{5} + \frac{1}{2} + \frac{1}{4} = \frac{3}{5} + \frac{1}{2} + \frac{1}{4}$ ________________________________

5. $\left(\frac{2}{7} + \frac{1}{5} + \frac{2}{3}\right) \bullet 1 = \left(\frac{2}{7} + \frac{1}{5} + \frac{2}{3}\right)$ ________________________________

6. $\frac{3}{4} + 0 = \frac{3}{4}$ ________________________________

7. $\frac{1}{2} + \frac{5}{6} = \frac{4}{3}$ ________________________________

8. $\frac{3}{8} \bullet \frac{1}{4} \bullet \frac{2}{3} \bullet \frac{1}{2} = \frac{3}{8} \bullet \frac{2}{3} \bullet \frac{1}{4} \bullet \frac{1}{2}$ ________________________________

9. $\frac{1}{4} \bullet \frac{5}{7} - \frac{2}{3} = \frac{1}{4} \bullet \frac{5}{7} - \frac{1}{4} \bullet \frac{2}{3}$ ________________________________

10. $\left(\frac{2}{15} \bullet \frac{5}{7}\right) \bullet 0 = 0$ ________________________________

B. Find the value of N in each expression

1. N + $\frac{1}{45} = \frac{1}{45}$

2. $\frac{1}{4} \bullet N \quad \bullet \frac{2}{3} = \frac{1}{4} \bullet \quad \frac{6}{7} \bullet \frac{2}{3}$

3. $\frac{2}{15} + \frac{12}{30} \quad + \frac{1}{5} = \frac{2}{15} + \quad N + \frac{1}{5}$

4. 0 + N = $\frac{5}{18}$

6. N $\bullet \quad \frac{6}{14} + \frac{2}{7} \quad = \frac{1}{6} \bullet \frac{6}{14} + \quad \frac{1}{6} \bullet \frac{2}{7}$

7. $\frac{8}{23} \bullet 1 = N$

8. $\frac{2}{9} + \frac{2}{3}$ = N

Summary

This lesson is about the properties of operations on rational numbers. The properties are useful because they simplify computations on rational numbers. These properties are true under the operations addition and multiplication. Note that for the Distributive Property of Multiplication over Subtraction, subtraction is considered part of addition. Think of subtraction as the addition of a negative rational number.

Lesson 10: Principal Roots and Irrational Numbers **Time: 2 hours**

Prerequisite Concepts: Set of rational numbers

About the Lesson:

This is an introductory lesson on irrational numbers, which may be daunting to students at this level. The key is to introduce them by citing useful examples.

Objectives:

In this lesson, you are expected to:

1. describe and define irrational numbers;
2. describe principal roots and tell whether they are rational or irrational;
3. determine between what two integers the square root of a number is;
4. estimate the square root of a number to the nearest tenth;
5. illustrate and graph irrational numbers (square roots) on a number line with and without appropriate technology.

Lesson Proper:

I. Activities

A. Take a look at the unusual wristwatch and answer the questions below.

1. Can you tell the time?
2. What time is shown in the wristwatch?
3. What do you get when you take the $\overline{1}$? $\overline{4}$? $\overline{9}$? $\overline{16}$?
4. How will you describe the result?
5. Can you take the exact value of $\overline{130}$?
6. What value could you get?

Taking the square root of a number is like doing the reverse operation of squaring a number. For example, both 7 and -7 are square roots of 49 since $7^2 = 49$ and $(-7)^2 = 49$. Integers such as 1, 4, 9, 16, 25 and 36 are called perfect squares. Rational numbers such as 0.16, $\frac{4}{100}$ and 4.84 are also, perfect squares. Perfect squares are numbers that have rational numbers as square roots. The square roots of perfect squares are rational numbers while the square roots of numbers that are not perfect squares are *irrational numbers*.

Any number that cannot be expressed as a quotient of two integers is an irrational number. The numbers $\sqrt{2}$, π, and the special number *e* are all irrational numbers. Decimal numbers that are non-repeating and non-terminating are irrational numbers.

B. Activity

Use the $\sqrt[n]{\ }$ button of a scientific calculator to find the following values:

1. $\sqrt[6]{64}$ 2. $\sqrt[4]{-16}$ 3. $\sqrt[3]{90}$

4. $\sqrt[5]{-3125}$ 5. $\sqrt{24}$

II. Questions to Ponder (Post-Activity Discussions)

Let us answer the questions in the opening activity.

1. Can you tell the time? *Yes*
2. What time is it in the wristwatch? *10:07*
3. What do you get when you take the $\overline{1}$? $\overline{4}$? $\overline{9}$? $\overline{16}$? *1, 2, 3, 4*
4. How will you describe the result? *They are all positive integers*.
5. Can you take the exact value of $\overline{130}$? *No*.
6. What value could you get? *Since the number is not a perfect square you could estimate the value to be between* $\sqrt{121}$ *and* $\sqrt{144}$*, which is about 11.4.*

Let us give the values asked for in Activity B. Using a scientific calculator, you probably obtained the following:

1. $\sqrt[6]{64}$ = *2*
2. $\sqrt[4]{-16}$ *Math Error, which means not defined*
3. $\sqrt[3]{90}$ = *4.481404747, which could mean non-terminating and non-repeating since the calculator screen has a limited size*
4. $\sqrt[5]{-3125}$ = *-5*
5. $\sqrt{24}$ = *4.898979486, which could mean non-terminating and non-repeating since the calculator screen has a limited size*

<u>On Principal n^{th} Roots</u>

Any number, say *a*, whose n^{th} power (*n*, a positive integer), is *b* is called the n^{th} root of b. Consider the following: $(-7)^2 = 49$, $2^4 = 16$ and $(-10)^3 = -1000$. This means that -7 is a 2^{nd} or square root of 49, 2 is a 4^{th} root of 16 and -10 is a 3^{rd} or cube root of -1000.

However, we are not simply interested in any n^{th} root of a number; we are more concerned about the principal n^{th} root of a number. **The principal n^{th} root of a positive number is the positive n^{th} root. The principal n^{th} root of a negative number is the negative n^{th} root if *n* is odd. If *n* is even and the number is negative, the principal n^{th} root is not defined. The notation for the principal n^{th} root of a number *b* is $\sqrt[n]{b}$. In this expression, *n* is the index and *b* is the radicand. The n^{th} roots are also called radicals.**

Classifying Principal n^{th} Roots as Rational or Irrational Numbers
To determine whether a principal root is a rational or irrational number, determine if the radicand is a perfect n^{th} power or not. If it is, then the root is rational. Otherwise, it is irrational.

Problem 1. Tell whether the principal root of each number is rational or irrational.

(a) $\sqrt[3]{225}$ (b) $\sqrt{0.04}$ (c) $\sqrt[5]{-111}$ (d) $\sqrt{10000}$ (e) $\sqrt[4]{625}$

Answers:
a) $\sqrt[3]{225}$ *is irrational*
(b) $\sqrt{0.04}$ *= 0.2 is rational*
(c) $\sqrt[5]{-111}$ *is irrational*
(d) $\sqrt{10000}$ *= 100 is rational*
(e) $\sqrt[4]{625}$ *= 5 is rational*

If a principal root is irrational, the best you can do for now is to give an estimate of its value. Estimating is very important for all principal roots that are not roots of perfect n^{th} powers.

Problem 2. The principal roots below are between two integers. Find the two closest such integers.

(a) $\sqrt{19}$ (b) $\sqrt[3]{101}$ (c) $\sqrt{300}$

Solution:
(a) $\sqrt{19}$

16 is a perfect integer square and 4 is its principal square root. 25 is the next perfect integer square and 5 is its principal square root. Therefore, $\sqrt{19}$ *is between 4 and 5.*

(b) $\sqrt[3]{101}$

64 is a perfect integer cube and 4 is its principal cube root. 125 is the next perfect integer cube and 5 is its principal cube root. Therefore, $\sqrt[3]{101}$ *is between 4 and 5.*

(c) $\sqrt{300}$

289 is a perfect integer square and 17 is its principal square root. 324 is the next perfect integer square and 18 is its principal square root. Therefore, $\sqrt{300}$ *is between 17 and 18.*

Problem 3. Estimate each square root to the nearest *tenth*.

(a) $\sqrt{40}$ (b) $\sqrt{12}$ (c) $\sqrt{175}$

Solution:
(a) $\sqrt{40}$

The principal root $\sqrt{40}$ *is between 6 and 7, principal roots of the two perfect squares 36 and 49, respectively. Now, take the square of 6.5, midway between 6 and 7. Computing,* $(6.5)^2 = 42.25$. *Since 42.25 > 40 then* $\sqrt{40}$ *is closer to 6 than to 7. Now, compute for the squares of numbers between 6 and 6.5:* $(6.1)^2 = 37.21$, $(6.2)^2 = 38.44$, $(6.3)^2 = 39.69$, *and* $(6.4)^2 = 40.96$. *Since 40 is close to 39.69 than to 40.96,* $\sqrt{40}$ *is approximately 6.3.*

(b) $\sqrt{12}$

The principal root $\sqrt{12}$ *is between 3 and 4, principal roots of the two perfect squares 9 and 16, respectively. Now take the square of 3.5, midway between 3 and 4. Computing* $(3.5)^2 = 12.25$. *Since 12.25 > 12 then* $\sqrt{12}$ *is closer to 3 than to 4. Compute for the squares of numbers between 3 and 3.5:* $(3.1)^2 = 9.61$, $(3.2)^2 = 10.24$, $(3.3)^2 = 10.89$, *and* $(3.4)^2 = 11.56$. *Since 12 is closer to 12.25 than to 11.56,* $\sqrt{12}$ *is approximately 3.5.*

(c) $\sqrt{175}$

The principal root $\sqrt{175}$ *is between 13 and 14, principal roots of the two perfect squares 169 and 196. The square of 13.5 is 182.25, which is greater than 175. Therefore,* $\sqrt{175}$ *is closer to 13 than to 14. Now:* $(13.1)^2 = 171.61$, $(13.2)^2 = 174.24$, $(13.3)^2 = 176.89$. *Since 175 is closer to 174.24 than to 176.89 then,* $\sqrt{175}$ *is approximately 13.2.*

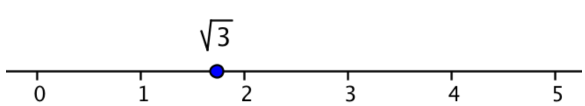
Problem 4. Locate and plot each square root on a number line.

(a) $\sqrt{3}$ (b) $\sqrt{21}$ (c) $\sqrt{87}$

Solution: You may use a program like Geogebra to plot the square roots on a number line.

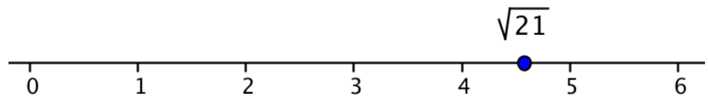
(a) $\sqrt{3}$

This number is between 1 and 2, principal roots of 1 and 4. Since 3 is closer to 4 than to 1, $\sqrt{3}$ is closer to 2. Plot $\sqrt{3}$ closer to 2.



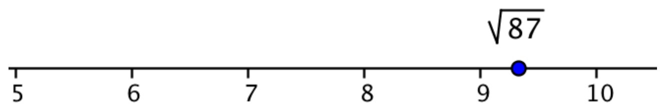
(b) $\sqrt{21}$

This number is between 4 and 5, principal roots of 16 and 25. Since 21 is closer to 25 than to 16, $\sqrt{21}$ is closer to 5 than to 4. Plot $\sqrt{21}$ closer to 5.



(c) $\overline{87}$

This number is between 9 and 10, principal roots of 81 and 100. Since 87 is closer to 81, then $\overline{87}$ is closer to 9 than to 10. Plot $\overline{87}$ closer to 9.



III. Exercises

A. Tell whether the principal roots of each number is rational or irrational.

1.	$\overline{400}$	6.	$\overline{13{,}689}$
2.	$\overline{64}$	7.	$\overline{1000}$
3.	$\overline{0.01}$	8.	$\overline{2.25}$
4.	$\overline{26}$	9.	$\overline{39}$
5.	$\overline{\frac{1}{49}}$	10.	$\overline{12.1}$

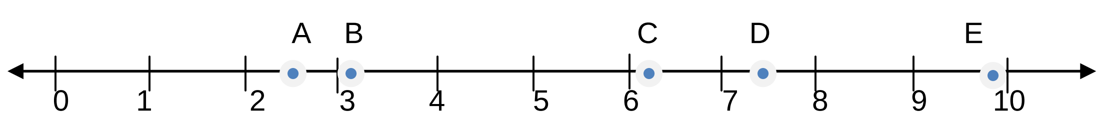
B. Between which two consecutive integers does the square root lie?

1.	$\overline{77}$	6.	$\overline{90}$
2.	$\overline{700}$	7.	$\overline{2045}$
3.	$\overline{243}$	8.	$\overline{903}$
4.	$\overline{444}$	9.	$\overline{1899}$
5.	$\overline{48}$	10.	$\overline{100000}$

C. Estimate each square root to the nearest *tenth* and plot on a number line.

1.	$\overline{50}$	6.	$\overline{250}$
2.	$\overline{72}$	7.	$\overline{5}$
3.	$\overline{15}$	8.	$\overline{85}$
4.	$\overline{54}$	9.	$\overline{38}$
5.	$\overline{136}$	10.	$\overline{101}$

D. Which point on the number line below corresponds to which square root?



1. $\overline{57}$ _______
2. $\overline{6}$ _______
3. $\overline{99}$ _______
4. $\overline{38}$ _______
5. $\overline{11}$ _______

Summary

In this lesson, you learned about irrational numbers and principal *n*th roots, particularly square roots of numbers. You learned to find two consecutive integers between which an irrational square root lies. You also learned how to estimate the square roots of numbers to the nearest tenth and how to plot the estimated square roots on a number line.

Lesson 11: The Absolute Value of a Number **Time: 1.5 hours**

Prerequisite Concepts: Set of real numbers

About the Lesson:

This lesson explains why a distance between two points, even if represented on a number line cannot be expressed as a negative number. Intuitively, the absolute value of a number may be thought of as the non-negative value of a number. The concept of absolute value is important to designate the magnitude of a measure such as *the temperature dropped by 23* (the absolute value) *degrees*. A similar concept is applied to profit vs loss, income against expense, and so on.

Objectives:

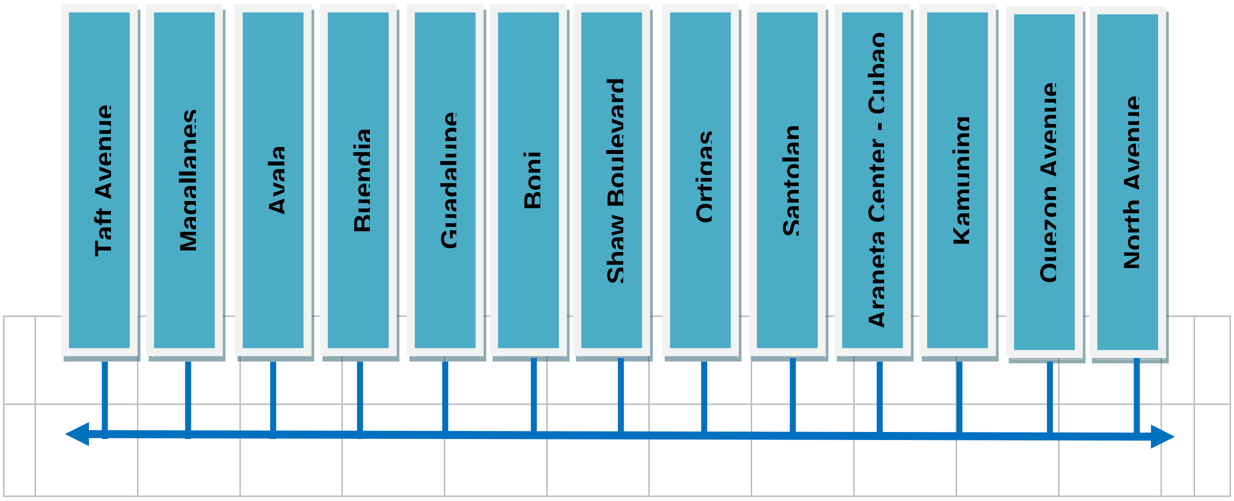
In this lesson, you are expected to describe and illustrate

a. the absolute value of a number on a number line.
b. the distance of the number from 0.

Lesson Proper:

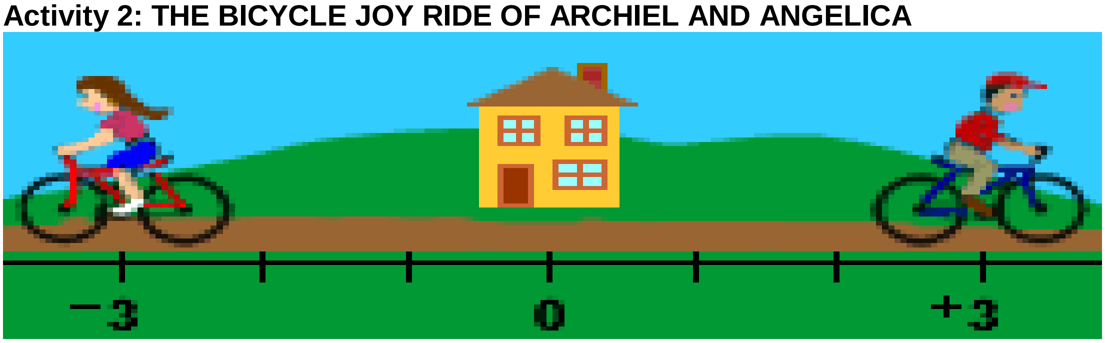
I. Activity 1: THE METRO MANILA RAIL TRANSIT (MRT) TOUR

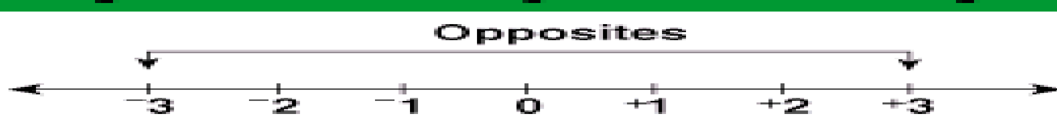
Suppose the MRT stations from Pasay City to Quezon City were on a straight line and were 500 meters apart from each other.



1. How far would the North Avenue station be from Taft Avenue?
2. What if Elaine took the MRT from North Avenue and got off at the last station? How far would she have travelled?
3. Suppose both Archie and Angelica rode the MRT at Shaw Boulevard and the former got off in Ayala while the latter in Kamuning. How far would each have travelled from the starting point to their destinations?
4. What can you say about the directions and the distances travelled by Archie and Angelica?

Activity 2: THE BICYCLE JOY RIDE OF ARCHIEL AND ANGELICA





Problem: Archie and Angelica were at Aloys' house. Angelica rode her bicycle 3 miles west of Aloys' house, and Archie rode his bicycle 3 miles east of Aloys' house. Who travelled a greater distance from Aloys' house – Archie or Angelica?

Questions To Ponder:

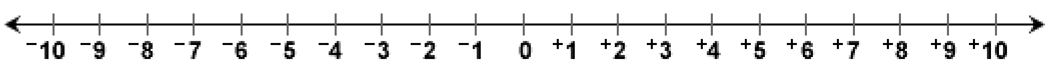
1. What subsets of real numbers are used in the problem? Represent the trip of Archie and Angelica to the house of Aloys using a number line.

2. What are opposite numbers on the number line? Give examples and show on the number line.
3. What does it mean for the same distance travelled but in opposite directions? How would you interpret using the numbers -3 and +3?
4. What can you say about the absolute value of opposite numbers say -5 and +5?
5. How can we represent the absolute value of a number? What notation can we use?

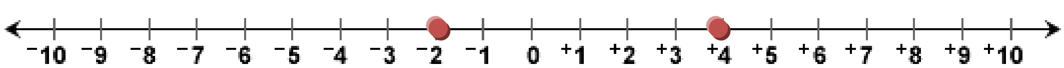
Important Terms to Remember
The following are terms that you must remember from this point on.

1. **Absolute Value –** of a number is the distance between that number and zero on the number line.
2. **Number Line –**is best described as a straight line which is extended in both directions as illustrated by arrowheads. A number line consists of three elements:
   a. set of positive numbers, and is located to the right of zero.
   b. set of negative numbers, and is located to the left of zero; and
   c. Zero.

Notations and Symbols
The absolute value of a number is denoted by two bars | |.
Let's look at the number line:



The absolute value of a number, denoted "| |" is the distance of the number from zero. This is why the absolute value of a number is never negative. In thinking about the absolute value of a number, one only asks "how far?" not "in which direction?" Therefore, the absolute value of 3 and of -3 is the same, which is 3 because both numbers have the same distance from zero.



Warning: The absolute-value notation is *bars*, not parentheses or brackets. Use the proper notation; the other notations do *not* mean the same thing.
It is important to note that the absolute value bars do NOT work in the same way as do parentheses. Whereas – (–3) = +3, this is NOT how it works for absolute value:

Problem: Simplify – | –3 |.

Solution: Given – | – 3 |, first find the absolute value of – 3.

– | –3 | = – (3)

Now take the negative of 3. Thus, :

– | –3 | = – (3) = **–3**

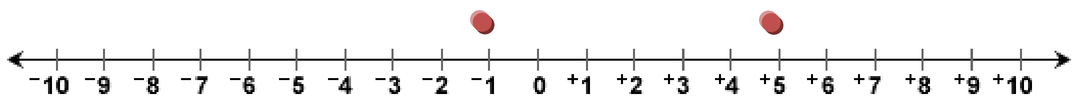
This illustrates that if you take the negative of the absolute value of a number, you will get a negative number for your answer.

II. Questons to Ponder(Post-Activity Discussion)

Let us answer the questions posed in Activity 2.

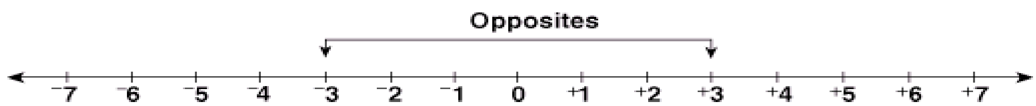
1. What subsets of real numbers are used in the problem? Represent the trip of Archie and Angelica to the house of Aloys using a number line.

The problem uses *<u>integers</u>. Travelling 3 miles west can be represented by -3 (pronounced negative 3). Travelling 3 miles east can be represented by +3 (pronounced positive 3). Aloys' house can be represented by the integer 0.*



2. What are opposite numbers on the number line? Give examples and show on the number line.

   Two integers that are the same distance from zero in opposite directions ***opposites***. *The integers $^{+}3$ and $^{-}3$ are opposites since they are each 3 zero.*



3. What does it mean for the same distance travelled but in opposite directions? How would you interpret using the numbers -3 and +3?
   The absolute value of a number is its distance from zero on the number line. The absolute value of +3 is 3, and the absolute value of -3 is 3.

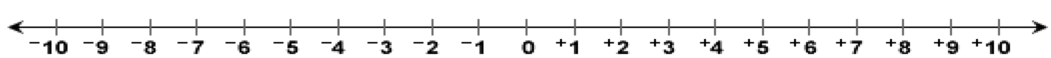
4. What can you say about the absolute value of opposite numbers say -5 and +5?
   Opposite numbers have the same absolute values.

5. How can we represent the absolute value of a number? What notation can we use?
   The symbol | | *is used for the absolute value of a number.*

III. Exercises

Carry out the following tasks. Write your answers on the spaces provided for each number.

1. Find the absolute value of $^{+}3$, $^{-}3$, $^{+}7$, $^{-}5$, $^{+}9$, $^{-}8$, $^{+}4$, $^{-}4$. You may refer to the number line below. What should you remember when we talk about the absolute value of a number?



Solution: $|^{+}3| = 3$ $|^{+}9| = 9$

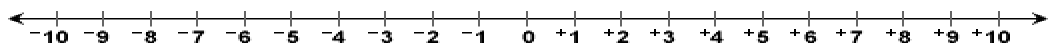
$|^{-}3| = 3$ $|^{-}8| = 8$

$|^{+}7| = 7$ $\quad$ $|^{+}4| = 4$

$|^{-}5| = 5$ $\quad$ $|^{-}4| = 4$

Remember that when we find the absolute value of a number, we are finding its distance from 0 on the number line. Opposite numbers have the same absolute value since they both have the same distance from 0. Also, you will notice that taking the absolute value of a number automatically means taking the positive value of that number.

2. Find the absolute value of: $^{+}11$, $^{-}9$, $^{+}14$, $^{-}10$, $^{+}17$, $^{-}19$, $^{+}20$, $^{-}20$. You may extend the number line below to help you solve this problem.



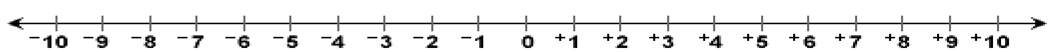
Solution: $|^{+}11| = 11$ $\quad$ $|^{+}17| = 17$

$|^{-}9| = 9$ $\quad$ $|^{-}19| = 19$

$|^{+}14| = 14$ $\quad$ $|^{+}20| = 20$

$|^{-}10| = 10$ $\quad$ $|^{-}20| = 20$

3. Use the number line below to find the value of N: $|N| = 5.1$



Solution: This problem asks us to find all numbers that are a distance of 5.1 units from zero on the number line. We let N represent all integers that satisfy this condition.

The number $^{+}5.1$ is 5.1 units from zero on the number line, and the number $^{-}5.1$ is also 5.1 units from zero on the number line. Thus both $^{+}5.1$ and $^{-}5.1$ satisfy the given condition.

4. When is the absolute value of a number equal to itself?

Solution:

When the value of the number is positive or zero.

5. Explain why the absolute value of a number is never negative. Give an example that will support your answer.

Solution: *Let* $|N| = -4$*. Think of a number that when you get the absolute value will give you a negative answer. There will be no solution since the distance of any number from 0 cannot be a negative quantity.*

Enrichment Exercises:

A. Simplify the following.

1. $|7.04|$
2. $|0|$
3. $|-\frac{2}{9}|$
4. $-|15 + 6|$
5. $|-2\sqrt{2}| - |-3\sqrt{2}|$

B. List at least two integers that can replace N such that.

1. $|N| = 4$
2. $|N| < 3$
3. $|N| > 5$
4. $|N| \leq 9$
5. $0 < |N| < 3$

C. Answer the following.

1. Insert the correct relation symbol(>, =, <): $|-7|$ ____ $|-4|$.
2. If $|x - 7| = 5$, what are the possible values of x?
3. If $|x| = \frac{2}{7}$, what are the possible values of x?
4. Evaluate the expression, $|x + y| - |y - x|$, if x = 4 and y = 7.
5. A submarine navigates at a depth of 50 meters below sea level while exactly above it; an aircraft flies at an altitude of 185 meters. What is the distance between the two carriers?

Summary:

In this lesson you learned about the absolute value of a number, that it is a distance from zero on the number line denoted by the notation |N|. This notation is used for the absolute value of an unknown number that satisfies a given condition. You also learned that a distance can never be a negative quantity and absolute value pertains to the magnitude rather than the direction of a number.