

LESSON 12: SUBSETS OF REAL NUMBERS

Time: 1.5 hours

Prerequisite Concepts: whole numbers and operations, set of integers, rational numbers, irrational numbers, sets and set operations, Venn diagrams

About the Lesson:

This lesson will intensify the study of mathematics since this requires a good understanding of the sets of numbers for easier communication. Classifying numbers is very helpful as it allows us to categorize what kind of numbers we are dealing with every day.

Objectives:

In this lesson, you are expected to :

2. Describe and illustrate the real number system.
3. Apply various procedures and manipulations on the different subsets of the set of real numbers.
 - a. Describe, represent and compare the different subsets of real number.
 - b. Find the union, intersection and complement of the set of real numbers and its subsets

Lesson Proper:

A. The Real Number System

I. Activity 1: Try to reflect on these . . .

It is difficult for us to realize that once upon a time there were no symbols or names for numbers. In the early days, primitive man showed how many animals he owned by placing an equal number of stones in a pile, or sticks in a row. Truly our number system evolved over hundreds of years.

Sharing Ideas! What do you think?

1. In what ways do you think did primitive man need to use numbers?
2. Why do you think he needed names or words to tell “how many”?
3. How did number symbols come about?
4. What led man to invent numbers, words and symbols?

Activity 2: LOOK AROUND!

Fifteen different words/partitions of numbers are hidden in this puzzle. How many can you find? Look up, down, across, backward, and diagonally. Figures are scattered around that will serve as clues to help you locate the mystery words.

N	A	F	R	A	C	T	I	O	N	S	I
S	P	B	A	C	C	D	Z	W	N	E	L
T	E	O	F	T	O	G	E	H	E	R	A
O	R	H	S	I	U	J	R	O	G	A	M
I	C	R	K	I	N	R	O	L	A	T	I
L	E	E	L	M	T	N	A	E	T	I	C
A	N	A	O	P	I	I	Q	L	I	O	E
R	T	L	R	S	N	T	V	U	V	N	D
U	I	N	T	E	G	E	R	E	E	A	A
T	I	R	R	A	T	I	O	N	A	L	I
A	N	O	N	I	N	T	E	G	E	R	S
N	N	U	M	N	U	M	B	E	R	S	S

Answer the following questions:

1. How many words in the puzzle were familiar to you?
2. What word/s have you encountered in your early years? Define and give examples.
3. What word/s is/are still strange to you?

Activity 3: Determine what set of numbers will represent the following situations:

1. Finding out how many cows there are in a barn
2. Corresponds to no more apples inside the basket
3. Describing the temperature in the North Pole
4. Representing the amount of money each member gets when P200 prize is divided among 3 members
5. Finding the ratio of the circumference to the diameter of a circle, denoted π (read "pi")

The set of numbers called **the real number system** consists of different partitions/**subsets** that can be represented graphically on a **number line**.

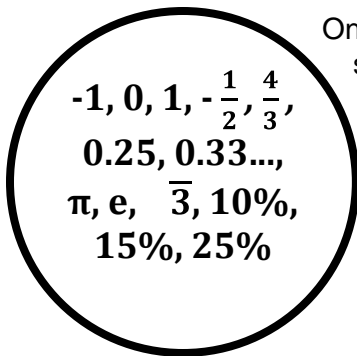
II. Questions to Ponder

Consider the activities done earlier and recall the different terms you encountered including the set of real numbers and together let us determine the various subsets. Let us go back to the first time we encountered the numbers...

Let's talk about the various subsets of real numbers.

Early Years...

1. What subset of real numbers do children learn at an early stage when they were just starting to talk? Give examples.



One subset is the **counting (or natural) numbers**. This subset includes all the numbers we use to count starting with "1" and so on. The subset would look like this: {1, 2, 3, 4, 5...}

In School at an Early Phase...

2. What do you call the subset of real numbers that includes zero (the number that represents nothing) and is combined with the subset of real numbers learned in the early years? Give examples.

Another subset is the **whole numbers**. This subset is exactly like the subset of counting numbers, with the addition of one extra number. This extra number is "0". The subset would look like this: {0, 1, 2, 3, 4...}

In School at Middle Phase...

3. What do you call the subset of real numbers that includes negative numbers (that came from the concept of "opposites" and specifically used in describing debt or below zero temperature) and is united with the whole numbers? Give examples.

A third subset is the **integers**. This subset includes all the whole numbers and their “opposites”. The subset would look like this: {... -4, -3, -2, -1, 0, 1, 2, 3, 4...}

Still in School at Middle Period...

4. What do you call the subset of real numbers that includes integers and non-integers and are useful in representing concepts like “**half a gallon of milk**”? Give examples.

The next subset is the **rational numbers**. This subset includes all numbers that “come to an end” or numbers that repeat and have a pattern. Examples of rational numbers are: 5.34, 0.131313..., $\frac{6}{7}$, $\frac{2}{3}$, 9

5. What do you call the subset of real numbers that is not a rational number but are physically represented like “**the diagonal of a square**”?

Lastly we have the set of **irrational numbers**. This subset includes numbers that cannot be exactly written as a decimal or fraction. Irrational numbers cannot be expressed as a ratio of two integers. Examples of irrational numbers are:

$$\sqrt{2}, \sqrt[3]{101}, \text{ and } \pi$$

Important Terms to Remember

The following are terms that you must remember from this point on.

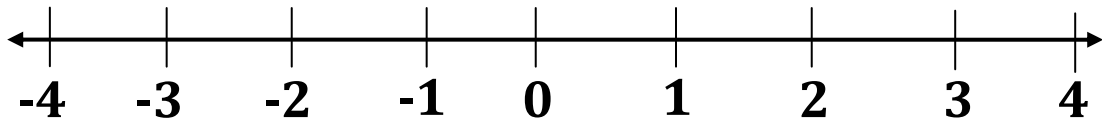
1. **Natural/Counting Numbers** – are the numbers we use in counting things, that is {1, 2, 3, 4, . . . }. The three dots, called ellipses, indicate that the pattern continues indefinitely.
2. **Whole Numbers** – are numbers consisting of the set of natural or counting numbers and zero.
3. **Integers** – are the result of the union of the set of whole numbers and the negative of counting numbers.
4. **Rational Numbers** – are numbers that can be expressed as a quotient $\frac{a}{b}$ of two integers. The integer **a** is the numerator while the integer **b**, which cannot be 0 is the denominator. This set includes fractions and some decimal numbers.
5. **Irrational Numbers** – are numbers that cannot be expressed as a quotient $\frac{a}{b}$ of two integers. Every irrational number may be represented by a decimal that neither repeats nor terminates.
6. **Real Numbers** – are any of the numbers from the preceding subsets. They can be found on the real number line. The union of rational numbers and irrational numbers is the set of real numbers.

7. **Number Line** – a straight line extended on both directions as illustrated by arrowheads and is used to represent the set of real numbers. On the real number line, there is a point for every real number and there is a real number for every point.

III. Exercises

- a. Locate the following numbers on the number line by naming the correct point.

$$\boxed{-2.66\dots, -1\frac{1}{2}, -0.25, \frac{3}{4}, \sqrt{2}, \sqrt[3]{11}}$$



- b. Determine the subset of real numbers to which each number belongs. Use a tick mark (\checkmark) to answer.

Number	Whole Number	Integer	Rational	Irrational
1. -86				
2. 34.74				
3. $\frac{4}{7}$				
4. $\sqrt{64}$				
5. $\sqrt{11}$				
6. -0.125				
7. $-\sqrt{81}$				
8. e				
9. -45.37				
10. -1.252525...				

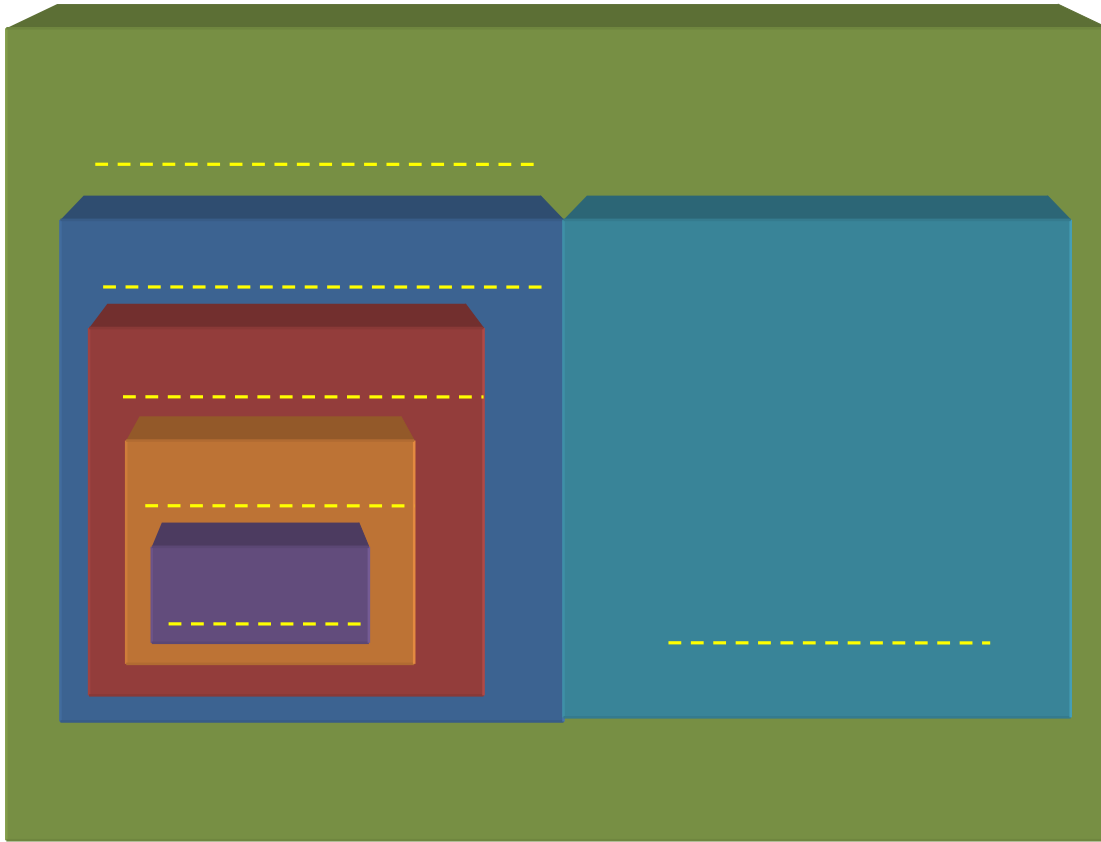
B. Points to Contemplate

It is interesting to note that the set of rational numbers and the set of irrational numbers are disjoint sets; that is their intersection is empty. In fact, they are complements of each other. The union of these two sets is the set of real numbers.

Exercises:

1. Based on the stated information, show the relationships among *natural/counting numbers*, *whole numbers*, *integers*, *rational numbers*, *irrational numbers* and

real numbers using the Venn diagram. Fill each broken line with its corresponding answer.



2. Answer the following questions on the space provided for each number.

a) Are all real numbers rational numbers? Prove your answer.

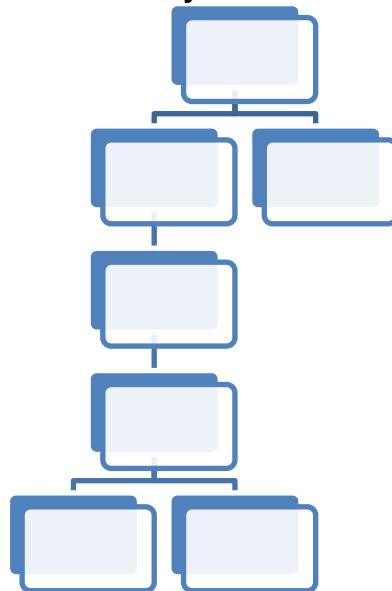
b) Are all rational numbers whole numbers? Prove your answer.

c) Are $-\frac{1}{4}$ and $-\frac{2}{5}$ negative integers? Prove your answer.

d) How is a rational number different from an irrational number?

e) How do natural numbers differ from whole numbers?

3. Complete the details in the Hierarchy Chart of the Set of Real Numbers.



THE REAL NUMBER SYSTEM

Summary

In this lesson, you learned different subsets of real numbers that enable you to name numbers in different ways. You also learned to determine the hierarchy and relationship of one subset to another that leads to the composition of the real number system using the Venn Diagram and Hierarchy Chart. You also learned that it was because of necessity that led man to invent number, words and symbols.

Lesson 13: Significant Digits and the Scientific Notation

OPTIONAL

Prerequisite Concepts: Rational numbers and powers of 10

About the Lesson:

This is a lesson on significant digits and the scientific notation combined. The use of significant digits and the scientific notation is often in the area of measures and in the natural sciences. The scientific notation simplifies the way we write very large and very small numbers. On the other hand, numerical data become more accurate when significant digits are taken into account.

Objectives:

In this lesson, you are expected to :

1. determine the significant digits in a given situation.
2. write very large and very small numbers in scientific notation

Lesson Proper:

I. A. Activity

The following is a list of numbers. The number of significant digits in each number is written in the parenthesis after the number.

234 (3)	0.0122 (3)
745.1 (4)	0.00430 (3)
6007 (4)	0.0003668 (4)
1.3×10^2 (2)	10000 (1)
7.50×10^{-7} (3)	1000. (4)
0.012300 (5)	2.222×10^{-3} (4)
100.0 (4)	8.004×10^5 (4)
100 (1)	6120. (4)
7890 (3)	120.0 (4)
4970.00 (6)	530 (2)

Describe what digits are not significant. _____

Important Terms to Remember

Significant digits are the digits in a number that express the precision of a measurement rather than its magnitude. The number of significant digits in a given measurement depends on the number of significant digits in the given data. In

calculations involving multiplication, division, trigonometric functions, for example, the number of significant digits in the final answer is equal to the least number of significant digits in any of the factors or data involved.

Rules for Determining Significant Digits

- A. All digits that are not zeros are significant.
For example: 2781 has 4 significant digits
82.973 has 5 significant digits
- B. Zeros may or may not be significant. Furthermore,
 - 1. Zeros appearing between nonzero digits are significant.
For example: 20.1 has 3 significant digits
79002 has 5 significant digits
 - 2. Zeros appearing in front of nonzero digits are not significant.
For example: 0.012 has 2 significant digits
0.0000009 has 1 significant digit
 - 3. Zeros at the end of a number and to the right of a decimal are significant digits. Zeros between nonzero digits and significant zeros are also significant.
For example: 15.0 has 3 significant digits
25000.00 has 7 significant digits
 - 4. Zeros at the end of a number but to the left of a decimal may or may not be significant. If such a zero has been measured or is the first estimated digit, it is significant. On the other hand, if the zero has not been measured or estimated but is just a place holder it is not significant. A decimal placed after the zeros indicates that they are significant
For example: 560000 has 2 significant digits
560000. has 6 significant digits

Significant Figures in Calculations

- 1. When multiplying or dividing measured quantities, round the answer to as many significant figures in the answer as there are in the measurement with the least number of significant figures.
- 2. When adding or subtracting measured quantities, round the answer to the same number of decimal places as there are in the measurement with the least number of decimal places.
For example:
 - a. $3.0 \times 20.536 = 61.608$

Answer: 61 since the least number of significant digits is 2, coming from 3.0

b. $3.0 + 20.536 = 23.536$

Answer: 23.5 since the addend with the least number of decimal places is 3.0

II. Questions to Ponder (Post-Activity Discussion)

Describe what digits are not significant. *The digits that are not significant are the zeros before a non-zero digit and zeros at the end of numbers without the decimal point.*

Problem 1. Four students weigh an item using different scales. These are the values they report:

- a. 30.04 g
- b. 30.0 g
- c. 0.3004 kg
- d. 30 g

How many significant digits are in each measurement?

Answer: *30.04 has 4 significant; 30.0 has 3 significant digits; 0.3004 has 4 significant digits; 30 has 1 significant digit*

Problem 2. Three students measure volumes of water with three different devices. They report the following results:

Device	Volume
Large graduated cylinder	175 mL
Small graduated cylinder	39.7 mL
Calibrated buret	18.16 mL

If the students pour all of the water into a single container, what is the total volume of water in the container? How many digits should you keep in this answer?

Answer: The total volume is 232.86 mL. Based on the measures, the final answer should be 232.9 mL.

On the Scientific Notation

The speed of light is 300 000 000 m/sec, quite a large number. It is cumbersome to write this number in full. Another way to write it is 3.0×10^8 . How about a very small number like 0.000 000 089? Like with a very large number, a very small number may be written more efficiently. 0.000 000 089 may be written as 8.9×10^{-8} .

Writing a Number in Scientific Notation

1. Move the decimal point to the right or left until after the first significant digit and copy the significant digits to the right of the first digit. If the number is a whole number and has no decimal point, place a decimal point after the first significant digit and copy the significant digits to its right.

For example, 300 000 000 has 1 significant digit, which is 3. Place a decimal point after 3.0

The first significant digit in 0.000 000 089 is 8 and so place a decimal point after 8, (8.9).

2. Multiply the adjusted number in step 1 by a power of 10, the exponent of which is the number of digits that the decimal point moved, positive if moved to the left and negative if moved to the right.

For example, 300 000 000 is written as 3.0×10^8 because the decimal point was moved past 8 places.

0.0 000 089 is written as 8.9×10^{-8} because the decimal point was moved 8 places to the right past the first significant digit 8.

III. Exercises

- A. Determine the number of significant digits in the following measurements. Rewrite the numbers with at least 5 digits in scientific notation.

- | | |
|----------------|-----------------------------|
| 1. 0.0000056 L | 6. 8207 mm |
| 2. 4.003 kg | 7. 0.83500 kg |
| 3. 350 m | 8. 50.800 km |
| 4. 4113.000 cm | 9. 0.0010003 m ³ |
| 5. 700.0 mL | 10. 8 000 L |

- B. a. Round off the following quantities to the specified number of significant figures.

1. 5 487 129 m to three significant figures
2. 0.013 479 265 mL to six significant figures
3. 31 947.972 cm² to four significant figures
4. 192.6739 m² to five significant figures

5. 786.9164 cm to two significant figures

b. Rewrite the answers in (a) using the scientific notation

C. Write the answers to the correct number of significant figures

1. $4.5 \times 6.3 \div 7.22$ _____
2. $5.567 \times 3.0001 \div 3.45$ _____
3. $(37 \times 43) \div (4.2 \times 6.0)$ _____
4. $(112 \times 20) \div (30 \times 63)$ _____
5. $47.0 \div 2.2$ _____

D. Write the answers in the correct number of significant figures

1. $5.6713 + 0.31 + 8.123$ _____
2. $3.111 + 3.11 + 3.1$ _____
3. $1237.6 + 23 + 0.12$ _____
4. $43.65 - 23.7$ _____
5. $0.009 - 0.005 + 0.013$ _____

E. Answer the following.

1. A runner runs the last 45m of a race in 6s. How many significant figures will the runner's speed have?
2. A year is 356.25 days, and a decade has exactly 10 years in it. How many significant figures should you use to express the number of days in two decades?
3. Which of the following measurements was recorded to 3 significant digits : 50 mL , 56 mL , 56.0 mL or 56.00 mL?
4. A rectangle measures 87.59 cm by 35.1 mm. Express its area with the proper number of significant figures in the specified unit: **a.** in cm^2
b. in mm^2
5. A 125 mL sample of liquid has a mass of 0.16 kg. What is the density of the liquid in g/mL?

Summary

In this lesson, you learned about significant digits and the scientific notation. You learned the rules in determining the number of significant digits. You also learned how to write very large and very small numbers using the scientific notation.

Lesson 14: More Problems Involving Real Numbers

Time: 1.5 hours

Pre-requisite Concepts: Whole numbers, Integers, Rational Numbers, Real Numbers, Sets

About the Lesson: This is the culminating lesson on real numbers. It combines all the concepts and skills learned in the past lessons on real numbers.

Objectives:

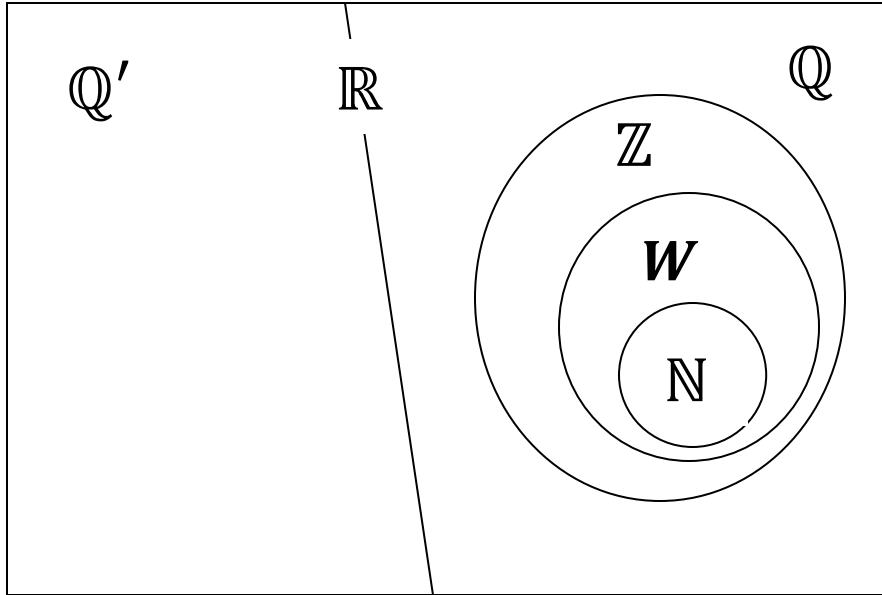
In this lesson, you are expected to:

1. Apply the set operations and relations to sets of real numbers
2. Describe and represent real-life situations which involve integers, rational numbers, square roots of rational numbers, and irrational numbers
3. Apply ordering and operations of real numbers in modeling and solving real-life problems

Lesson Proper:

Recall how the set of real numbers was formed and how the operations are performed. Numbers came about because people needed and learned to count. The set of counting numbers was formed. To make the task of counting easier, addition came about. Repeated addition then got simplified to multiplication. The set \mathbb{N} of counting numbers is closed under both the operations of addition and multiplication. When the need to represent zero arose, the set \mathbf{W} of whole numbers was formed. When the operation of subtraction began to be performed, the \mathbf{W} was extended to the set \mathbb{Z} or integers. \mathbb{Z} is closed under the operations of addition, multiplication and subtraction. The introduction of division needed the expansion of \mathbb{Z} to the set \mathbb{Q} of rational numbers. \mathbb{Q} is closed under all the four arithmetic operations of addition, multiplication, subtraction and division. When numbers are used to represent measures of length, the set \mathbb{Q} or rational numbers no longer sufficed. Hence, the set \mathbb{R} of real numbers came to be the field where properties work.

The above is a short description of the way the set of real numbers was built up to accommodate applications to counting and measurement and performance of the four arithmetic operations. We can also explore the set of real numbers by dissection – beginning from the big set, going into smaller subsets. We can say that \mathbb{R} is the set of all decimals (positive, negative and zero). The set \mathbb{Q} includes all the decimals which are repeating (we can think of terminating decimals as decimals in which all the digits after a finite number of them are zero). The set \mathbb{Z} comprises all the decimals in which the digits to the right of the decimal point are all zero. This view gives us a clearer picture of the relationship among the different subsets of \mathbb{R} in terms of inclusion.



We know that the n th root of any number which is not the n th power of a rational number is irrational. For instance, $\sqrt{2}$, $\sqrt{5}$, and $\sqrt[3]{9}$ are irrational.

Example 1. Explain why $\sqrt[3]{2}$ is irrational.

We use an argument called an indirect proof. This means that we will show why $\sqrt[3]{2}$ becoming rational will lead to an absurd conclusion. What happens if $\sqrt[3]{2}$ is rational? Because \mathbb{Q} is closed under multiplication and $\frac{1}{3}$ is rational, then $\sqrt[3]{2} \times \frac{1}{3}$ is rational. However, $\sqrt[3]{2} \times \frac{1}{3} = \sqrt[3]{2}$, which we know to be irrational. This is an absurdity. Hence we have to conclude that $\sqrt[3]{2}$ must be irrational.

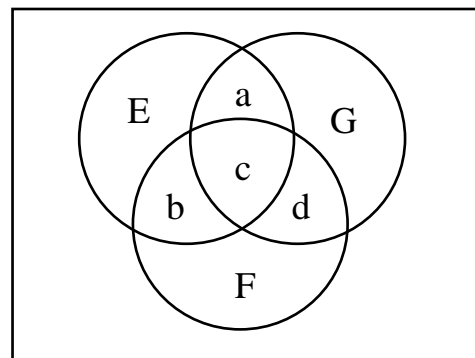
Example 2. A deep-freeze compartment is maintained at a temperature of 12°C below zero. If the room temperature is 31°C , how much warmer is the room temperature than the temperature in the deep-freeze compartment.

Get the difference between room temperature and the temperature inside the deep-freeze compartment

$31 - (-12) = 43$. Hence, room temperature is 43°C warmer than the compartment.

Example 3. Hamming Code

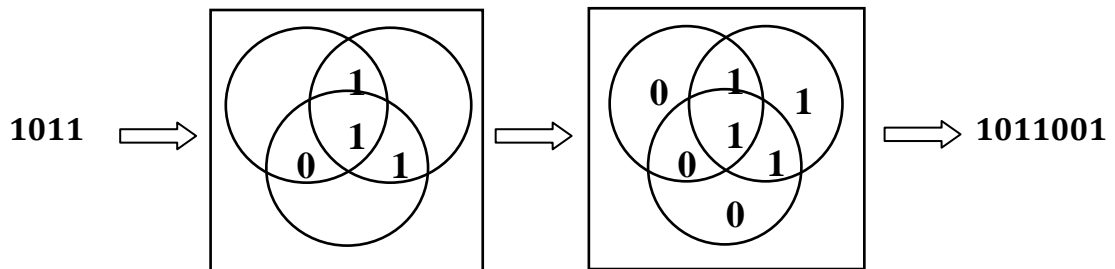
A mathematician, Richard Hamming developed an error detection code to determine if the information sent electronically is transmitted correctly. Computers store information using bits (binary digits, that is, a 0 or a 1). For example, 1011 is a four-bit code.



Hamming uses a Venn diagram with three “sets” as follows:

1. The digits of the four-bit code are placed in regions a, b, c, and d, in this order.
2. Three additional digits of 0's and 1's are put in the regions E, F, and G so that each “set” has an even number of 1's.
3. The code is then extended to a 7-bit code using (in order) the digits in the regions a, b, c, d, E, F, G.

For example, the code 1011 is encoded as follows:



Example 4. Two students are vying to represent their school in the regional chess competition. Felix won 12 of the 17 games he played this year, while Rommel won 11 of the 14 games he played this year. If you were the principal of the school, which student would you choose? Explain.

The Principal will likely use fractions to get the winning ratio or percentage of each player. Felix has a $\frac{12}{17}$ winning ratio, while Rommel has a $\frac{11}{14}$ winning ratio. Since $\frac{11}{14} > \frac{12}{17}$, Rommel will be a logical choice.

Example 5. A class is having an election to decide whether they will go on a fieldtrip. They will have a fieldtrip if more than 50% of the class will vote Yes. Assume that every member of the class will vote. If 34% of the girls and 28% of the boys will vote Yes, will the class go on a fieldtrip? Explain.

Although $34 + 28 = 64 > 50$, less than half of the girls and less than half of the boys voted Yes. This means that less than half all students voted Yes.

Example 6. A sale item was marked down by the same percentage for three years in a row. After two years the item was 51% off the original price. By how much was the price off the original price in the first year?

Since the price after 2 years is 51% off the original price, this means that the price is then 49% of the original. Since the percentage ratio must be multiplied to the original price twice (one per year), and $0.7 \times 0.7 = 0.49$, then the price per year is 70% of the price in the preceding year. Hence the discount is 30% off the original.

Exercises:

1. The following table shows the mean temperature in Moscow by month from 2001 to 2011

January	-6.4°C	May	13.7°C	September	12.6°C
February	-7.6°C	June	16.9°C	October	6.0°C
March	-0.8°C	July	21.0°C	November	0.5°C
April	6.8°C	August	18.4°C	December	-4.9°C

Plot each temperature point on the number line and list from lowest to highest.

2. Below are the ingredients for chocolate oatmeal raisin cookies. The recipe yields 32 cookies. Make a list of ingredients for a batch of 2 dozen cookies.

1 ½ cups all-purpose flour
1 tsp baking soda
1 tsp salt
1 cup unsalted butter
¾ cup light-brown sugar
¾ cup granulated sugar
2 large eggs
1 tsp vanilla extract
2 ½ cups rolled oats
1 ½ cups raisins
12 ounces semi-sweet chocolate chips

3. In high-rise buildings, floors are numbered in increasing sequence from the ground-level floor to second, third, etc, going up. The basement immediately below the ground floor is usually labeled B1, the floor below it is B2, and so on. How many floors does an elevator travel from the 39th floor of a hotel to the basement parking at level B6?
4. A piece of ribbon 25 m long is cut into pieces of equal length. Is it possible to get a piece with irrational length? Explain.
5. Explain why $5 + \sqrt{3}$ is irrational. (See Example 1.)

Lesson 15: Measurement and Measuring Length

Time: 2.5 hours

Prerequisite Concepts: Real Numbers and Operations

About the Lesson:

This is a lesson on the English and Metric System of Measurement and using these systems to measure length. Since these systems are widely used in our community, a good grasp of this concept will help you be more accurate in dealing with concepts involving length such as distance, perimeter and area.

Objective

At the end of the lesson, you should be able to:

1. Describe what it means to measure;
2. Describe the development of measurement from the primitive to the present international system of unit;
3. Estimate or approximate length;
4. Use appropriate instruments to measure length;
5. Convert length measurement from one unit to another, including the English system;
6. Solve problems involving length, perimeter and area.

Lesson Proper

A.

I. Activity:

Instructions: Determine the dimension of the following using only parts of your arms. Record your results in the table below. Choose a classmate and compare your results.

	SHEET OF INTERMEDIATE PAPER		TEACHER'S TABLE		CLASSROOM	
	Length	Width	Length	Width	Length	Width
Arm part used*						
Measurement						
Comparison to: (classmate's name)						

* For the arm part, please use any of the following only: the palm, the handspan and the forearm length

Important Terms to Remember:

>palm – the width of one's hand excluding the thumb

> handspan – the distance from the tip of the thumb to the tip of the little finger of one's hand with fingers spread apart.

> forearm length – the length of one's forearm: the distance from the elbow to the tip of the middle finger.

Answer the following questions:

1. What was your reason for choosing which arm part to use? Why?
2. Did you experience any difficulty when you were doing the actual measuring?
3. Were there differences in your data and your classmate's data? Were the differences significant? What do you think caused those differences?

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the opening activity:

1. What is the appropriate arm part to use in measuring the length and width of the sheet of paper? of the teacher's table? Of the classroom? What was your reason for choosing which arm part to use? Why?

- While all of the units may be used, there are appropriate units of measurement to be used depending on the length you are trying to measure.
- For the sheet of paper, the palm is the appropriate unit to use since the handspan and the forearm length is too long.
- For the teacher's table, either the palm or the handspan will do but the forearm length might be too long to get an accurate measurement.
- For the classroom, the palm and handspan may be used but you may end up with a lot of repetitions. The best unit to use would be the forearm length.

2. Did you experience any difficulty when you were doing the actual measuring?

The difficulties you may have experienced might include having to use too many repetitions.

3. Were there differences in your data and your classmate's data? Were the differences significant? What do you think caused those differences?

If you and your partner vary a lot in height, then chances are your forearm length, handspan and palm may also vary, leading to different measurements of the same thing.

History of Measurement

One of the earliest tools that human beings invented was the unit of measurement. In olden times, people needed measurement to determine how long or wide things are; things they needed to build their houses or make their clothes. Later, units of measurement were used in trade and commerce. In the 3rd century BC Egypt, people used their body parts to determine measurements of things; the same body parts that you used to measure the assigned things to you.

The forearm length, as described in the table below, was called a cubit. The handspan was considered a half cubit while the palm was considered 1/6 of a cubit. Go ahead, check out how many handspans your forearm length is. The Egyptians came up with these units to be more accurate in measuring different lengths.

However, using these units of measurement had a disadvantage. Not everyone had the same forearm length. Discrepancies arose when the people started comparing their measurements to one another because measurements of the same thing differed, depending on who was measuring it. Because of this, these units of measurement are called non-standard units of measurement which later on evolved into what is now the inch, foot and yard, basic units of length in the English system of measurement.

III. Exercise:

1. Can you name other body measurements which could have been used as a non-standard unit of measurement? Do some research on other non-standard units of measurement used by people other than the Egyptians.
2. Can you relate an experience in your community where a non-standard unit of measurement was used?

B.

I. Activity

Instructions: Determine the dimension of the following using the specified English units only. Record your results in the table below. Choose a classmate and compare your results.

	SHEET OF INTERMEDIATE PAPER		TEACHER'S TABLE		CLASSROOM	
	Length	Width	Length	Width	Length	Width
Arm part used*						
Measurement						
Comparison to: (classmate's name)						

For the unit used, choose which of the following SHOULD be used: inch or foot.

Answer the following questions:

1. What was your reason for choosing which unit to use? Why?
2. Did you experience any difficulty when you were doing the actual measuring?
3. Were there differences in your data and your classmate's data? Were the differences as big as the differences when you used non-standard units of measurement? What do you think caused those differences?

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

1. What was your reason for choosing which unit to use? Why?
 - For the sheet of paper, the appropriate unit to use is inches since its length and width might be shorter than a foot.
 - For the table and the classroom, a combination of both inches and feet may be used for accuracy and convenience of not having to deal with a large number.
2. What difficulty, if any, did you experience when you were doing the actual measuring?
3. Were there differences in your data and your classmate's data? Were the differences as big as the differences when you used non-standard units of measurement? What do you think caused those differences?
 - If you and your partner used the steel tape correctly, both your data should have little or no difference at all. The difference should not be as big or as significant as the difference when non-standard units of measurement were

used. The slight difference might be caused by how accurately you tried to measure each dimension or by how you read the ticks on the steel tape. In doing actual measurement, a margin of error should be considered.

History of Measurement (Continued)

As mentioned in the first activity, the inch, foot and yard are said to be based on the cubit. They are the basic units of length of the English System of Measurement, which also includes units for mass, volume, time, temperature and angle. Since the inch and foot are both units of length, each can be converted into the other. Here are the conversion factors, as you may recall from previous lessons:

$$1 \text{ foot} = 12 \text{ inches}$$

$$1 \text{ yard} = 3 \text{ feet}$$

For long distances, the mile is used:

$$1 \text{ mile} = 1,760 \text{ yards} = 5,280 \text{ feet}$$

Converting from one unit to another might be tricky at first, so an organized way of doing it would be a good starting point. As the identity property of multiplication states, the product of any value and 1 is the value itself. Consequently, dividing a value by the same value would be equal to one. Thus, dividing a unit by its equivalent in another unit is equal to 1. For example:

$$1 \text{ foot} / 12 \text{ inches} = 1$$

$$3 \text{ feet} / 1 \text{ yard} = 1$$

These conversion factors may be used to convert from one unit to another. Just remember that you're converting from one unit to another so cancelling same units would guide you in how to use your conversion factors. For example:

1. Convert 36 inches into feet:

$$36 \cancel{\text{ inches}} \times \frac{1 \text{ foot}}{12 \cancel{\text{ inches}}} = \mathbf{3 \text{ feet}}$$

2. Convert 2 miles into inches:

$$2 \cancel{\text{ miles}} \times \frac{5280 \cancel{\text{ feet}}}{1 \cancel{\text{ mile}}} \times \frac{12 \text{ inches}}{1 \cancel{\text{ foot}}} = \frac{2 \times 5280 \times 12}{1 \times 1} \text{ inches} = \mathbf{126,720 \text{ inches}}$$

Again, since the given measurement was multiplied by conversion factors which are equal to 1, only the unit was converted but the given length was not changed.

Try it yourself.

III. Exercise:

Convert the following lengths into the desired unit:

1. Convert 30 inches to feet
2. Convert 130 yards to inches
3. Sarah is running in a 42-mile marathon. How many more feet does Sarah need to run if she has already covered 64,240 yards?

C.

I. Activity:

Answer the following questions:

1. When a Filipina girl is described as 1.7 meters tall, would she be considered tall or short? How about if the Filipina girl is described as 5 ft, 7 inches tall, would she be considered tall or short?

2. Which particular unit of height were you more familiar with? Why?

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

1. When a Filipina girl is described as 1.7 meters tall, would she be considered tall or short? How about if the Filipina girl is described as 5 ft, 7 inches tall, would she be considered tall or short?

➤ Chances are, you would find it difficult to answer the first question. As for the second question, a Filipina girl with a height of 5 feet, 7 inches would be considered tall by Filipino standards.

2. Which particular unit of height were you more familiar with? Why?

➤ Again, chances are you would be more familiar with feet and inches since feet and inches are still being widely used in measuring and describing height here in the Philippines.

History of Measurement (Continued)

The English System of Measurement was widely used until the 1800s and the 1900s when the Metric System of Measurement started to gain ground and became the most used system of measurement worldwide. First described by Belgian Mathematician Simon Stevin in his booklet, *De Thiende* (The Art of Tenths) and proposed by English philosopher, John Wilkins, the Metric System of Measurement was first adopted by France in 1799. In 1875, the General Conference on Weights and Measures (*Conférence générale des poids et mesures* or *CGPM*) was tasked to define the different measurements. By 1960, CGPM released the International System of Units (SI) which is now being used by majority of the countries with the biggest exception being the United States of America. Since our country used to be a colony of the United States, the Filipino people were schooled in the use of the English instead of the Metric System of Measurement. Thus, the older generation of Filipinos is more comfortable with English System rather than the Metric System although the Philippines have already adopted the Metric System as its official system of measurement.

The Metric System of Measurement is easier to use than the English System of Measurement since its conversion factors would consistently be in the decimal system, unlike the English System of Measurement where units of lengths have different conversion factors. Check out the units used in your steep tape measure, most likely they are inches and centimeters. The base unit for length is the meter and units longer or shorter than the meter would be achieved by adding prefixes to the base unit. These prefixes may also be used for the base units for mass, volume, time and other measurements. Here are the common prefixes used in the Metric System:

PREFIX	SYMBOL	FACTOR
tera	T	x 1,000,000,000,000
giga	G	x 1,000,000,000
mega	M	x 1,000,000
kilo	k	x 1,000
hecto	h	x 100
deka	da	x 10

deci	d	x 1/10
centi	c	x 1/100
milli	m	x 1/1,000
micro	μ	x 1/1,000,000
nano	n	x 1/1,000,000,000

For example:

1 kilometer = 1,000 meters

1 millimeter = 1/1,000 meter or 1,000 millimeters = 1 meter

These conversion factors may be used to convert from big to small units or vice versa. For example:

1. Convert 3 km to m:

$$3 \cancel{\text{km}} \times \frac{1,000 \text{ m}}{1 \cancel{\text{km}}} = 3,000\text{m}$$

2. Convert 10 mm to m:

$$10 \cancel{\text{mm}} \times \frac{1 \text{ m}}{1000 \cancel{\text{mm}}} = \frac{1}{100} \text{ or } 0.01\text{m}$$

As you can see in the examples above, any length or distance may be measured using the appropriate English or Metric units. In the question about the Filipina girl whose height was expressed in meters, her height can be converted to the more familiar feet and inches. So, in the Philippines where the official system of measurements is the Metric System yet the English System continues to be used, or as long as we have relatives and friends residing in the United States, knowing how to convert from the English System to the Metric System (or vice versa) would be useful. The following are common conversion factors for length:

1 inch = 2.54 cm

3.3 feet ≈ 1 meter

For example:

Convert 20 inches to cm:

$$20 \cancel{\text{in}} \times \frac{2.54 \text{ cm}}{1 \cancel{\text{in}}} = 50.8\text{cm}$$

III. Exercise:

1. Using the tape measure, determine the length of each of the following in cm. Convert these lengths to meters.

	PALM	HANDSPAN	FOREARM LENGTH
Centimeters			
Meters			

2. Using the data in the table above, estimate the lengths of the following without using the steel tape measure or ruler:

	BALLPEN	LENGTH OF WINDOW PANE	LENGTH OF YOUR FOOT FROM THE TIP OF YOUR HEEL TO THE TIP OF YOUR TOES	HEIGHT OF THE CHALK BOARD	LENGTH OF THE CHALK BOARD
NON-STANDARD UNIT					
METRIC UNIT					

3. Using the data from table 1, convert the dimensions of the sheet of paper, teacher's table and the classroom into Metric units. Recall past lessons on perimeter and area and fill in the appropriate columns:

	SHEET OF INTERMEDIATE PAPER				TEACHER'S TABLE				CLASSROOM			
	Length	Width	Perimeter	Area	Length	Width	Perimeter	Area	Length	Width	Perimeter	Area
English units												
Metric Units												

4. Two friends, Zale and Enzo, run in marathons. Zale finished a 21-km marathon in Cebu while Enzo finished a 15-mile marathon in Los Angeles. Who between the two ran a longer distance? By how many meters?

5. Georgia wants to fence her square garden, which has a side of 20 feet, with two rows of barb wire. The store sold barb wire by the meter at P12/meter. How much money will Georgia need to buy the barb wire she needs?

5. A rectangular room has a floor area of 32 square meters. How many tiles, each measuring 50 cm x 50 cm, are needed to cover the entire floor?

Summary

In this lesson, you learned: 1) that ancient Egyptians used units of measurement based on body parts such as the cubit and the half cubit. The cubit is the length of the forearm from the elbow to the tip of the middle finger; 2) that the inch and foot, the units for length of the English System of Measurement, are believed to be based on the cubit; 3) that the Metric System of Measurement became the dominant system in the 1900s and is now used by most of the countries with a few exceptions, the biggest exception being the United States of America; 4) that it is appropriate to

use short base units of length for measuring short lengths and long units of lengths to measure long lengths or distances; 5) how to convert common English units of length into other English units of length using conversion factors; 6) that the Metric System of Measurement is based on the decimal system and is therefore easier to use; 7) that the Metric System of Measurement has a base unit for length (meter) and prefixes to signify long or short lengths or distances; 8) how to estimate lengths and distances using your arm parts and their equivalent Metric lengths; 9) how to convert common Metric units of length into other Metric units of length using the conversion factors based on prefixes; 10) how to convert common English units of length into Metric units of length (and vice versa) using conversion factors; 11) how to solve length, perimeter and area problems using English and Metric units.

Lesson 16: Measuring Weight/Mass and Volume

Time: 2.5 hours

Prerequisite Concepts: Basic concepts of measurement, measurement of length

About the Lesson:

This is a lesson on measuring volume & mass/weight and converting its units from one to another. A good grasp of this concept is essential since volume & weight are commonplace and have practical applications.

Objectives:

At the end of the lesson, you should be able to:

7. estimate or approximate measures of weight/mass and volume;
8. use appropriate instruments to measure weight/mass and volume;
9. convert weight/mass and volume measurements from one unit to another, including the English system;
10. Solve problems involving weight/mass and volume/capacity.

Lesson Proper

A.

I. Activity:

Read the following narrative to help you review the concept of volume.

Volume

Volume is the amount of space an object contains or occupies. The volume of a container is considered to be the capacity of the container. This is measured by the number of cubic units or the amount of fluid it can contain and not the amount of space the container occupies. The base SI unit for volume is the cubic meter (m^3). Aside from cubic meter, another commonly used metric unit for volume of solids is the cubic centimeter (cm^3 or cc) while the commonly used metric units for volume of fluids are the liter (L) and the milliliter (mL).

Hereunder are the volume formulae of some regularly-shaped objects:

Cube: Volume = edge x edge x edge ($V = e^3$)

Rectangular prism: Volume = length x width x height ($V = lwh$)

Triangular prism: Volume = $\frac{1}{2}$ x base of the triangular base x height of the triangular base x Height of the prism

$$(V = (\frac{1}{2}bh)H)$$

Cylinder: Volume = π x (radius)² x height of the cylinder ($V = \pi r^2h$)

Other common regularly-shaped objects are the different pyramids, the cone and the sphere. The volumes of different pyramids depend on the shape of its base. Here are their formulae:

Square-based pyramids: Volume = $\frac{1}{3}$ x (side of base)² x height of pyramid ($V = \frac{1}{3} s^2h$)

Rectangle-based pyramid: Volume = $\frac{1}{3}$ x length of the base x width of the base x height of pyramid ($V = \frac{1}{3} lwh$)

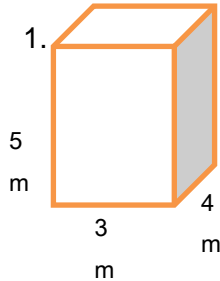
Triangle-based pyramid: Volume = $\frac{1}{3}$ x $\frac{1}{2}$ x base of the triangle x height of the triangle x Height of the pyramid

$$(V = \frac{1}{3}(\frac{1}{2}bh)H)$$

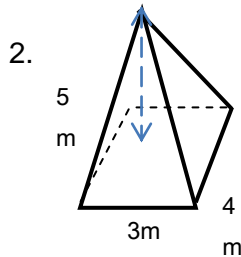
Cone: Volume = $\frac{1}{3} \times \pi \times (\text{radius})^2 \times \text{height}$

Sphere: Volume = $\frac{4}{3} \times \pi \times (\text{radius})^3$ ($V = \frac{4}{3} \pi r^3$)

Here are some examples:



$$V = lwh = 3 \text{ m} \times 4 \text{ m} \times 5 \text{ m} \\ = (3 \times 4 \times 5) \times (\text{m} \times \text{m} \times \text{m}) = 60 \text{ m}^3$$



$$V = \frac{1}{3} lwh = \frac{1}{3} \times 3 \text{ m} \times 4 \text{ m} \times 5 \text{ m} \\ = (\frac{1}{3} \times 3 \times 4 \times 5) \times (\text{m} \times \text{m} \times \text{m}) = 20 \text{ m}^3$$

Answer the following questions:

1. Cite a practical application of volume.
2. What do you notice about the parts of the formulas that have been underlined? Come up with a general formula for the volume of all the given prisms and for the cylinder.
3. What do you notice about the parts of the formulas that have been shaded? Come up with a general formula for the volume of all the given pyramids and for the cone.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the opening activity:

1. Cite a practical application of volume.
Volume is widely used from baking to construction. Baking requires a degree of precision in the measurement of the ingredients to be used thus measuring spoons and cups are used. In construction, volume is used to measure the size of a room, the amount of concrete needed to create a specific column or beam or the amount of water a water tank could hold.
2. What do you notice about the parts of the formulas that have been underlined? Come up with a general formula for the volume of all the given prisms and for the cylinder.
The formulas that have been underlined are formulas for area. The general formula for the volume of the given prisms and cylinder is just the area of the base of the prisms or cylinder times the height of the prism or cylinder ($V = A_{\text{base}}h$).

3. What do you notice about the parts of the formulas that have been shaded? Come up with a general formula for the volume of all the given pyramids and for the cone.

The formulas that have been shaded are formulas for the volume of prisms or cylinders. The volume of the given pyramids is just 1/3 of the volume of a prism whose base and height are equal to that of the pyramid while the formula for the cone is just 1/3 of the volume of a cylinder with the same base and height as the cone ($V = 1/3 V_{prism \text{ or } cylinder}$).

III. Exercise:

Instructions: Answer the following items. Show your solution.

1. How big is a Toblerone box (triangular prism) if its triangular side has a base of 3 cm and a height of 4.5 cm and the box's height is 25 cm?
2. How much water is in a cylindrical tin can with a radius of 7 cm and a height of 20 cm if it is only a quarter full?
3. Which of the following occupies more space, a ball with a radius of 4 cm or a cube with an edge of 60 mm?

B.

I. Activity

Materials Needed:

Ruler / Steel tape measure

Different regularly-shaped objects (brick, cylindrical drinking glass, balikbayan box)

Instructions: Determine the dimension of the following using the specified metric units only. Record your results in the table below and compute for each object's volume using the unit used to measure the object's dimensions. Complete the table by expressing/converting the volume using the specified units.

		BRICK			DRINKING GLASS		BALIKBAYAN BOX			CLASSROOM		
		Length	Width	Height	Radius	Height	Length	Width	Height	Length	Width	Height
Unit used*												
Measurement												
Volume	cm ³											
	m ³											
	in ³											
	ft ³											

For the unit used, choose ONLY one: centimeter or meter.

Answer the following questions:

1. What was your reason for choosing which unit to use? Why?
2. How did you convert the volume from cc to m³ or vice versa?
3. How did you convert the volume from cc to the English units for volume?

Volume (continued)

The English System of Measurement also has its own units for measuring volume or capacity. The commonly used English units for volume are cubic feet (ft^3) or cubic inches (in^3) while the commonly used English units for fluid volume are the pint, quart or gallon. Recall from the lesson on length and area that while the Philippine government has mandated the use of the Metric system, English units are still very much in use in our society so it is an advantage if we know how to convert from the English to the Metric system and vice versa. Recall as well from the previous lesson on measuring length that a unit can be converted into another unit using conversion factors. Hereunder are some of the conversion factors which would help you convert given volume units into the desired volume units:

$$1 \text{ m}^3 = 1 \text{ million cm}^3$$

$$1 \text{ ft}^3 = 1,728 \text{ in}^3$$

$$1 \text{ in}^3 = 16.4 \text{ cm}^3$$

$$1 \text{ m}^3 = 35.3 \text{ ft}^3$$

$$1 \text{ gal} = 3.79 \text{ L}$$

$$1 \text{ gal} = 4 \text{ quarts}$$

$$1 \text{ quart} = 2 \text{ pints}$$

$$1 \text{ pint} = 2 \text{ cups}$$

$$1 \text{ cup} = 16 \text{ tablespoons}$$

$$1 \text{ tablespoon} = 3 \text{ teaspoons}$$

Since the formula for volume only requires length measurements, another alternative to converting volume from one unit to another is to convert the object's dimensions into the desired unit before solving for the volume.

For example:

1. How much water, in cubic centimeters, can a cubical water tank hold if it has an edge of 3 meters?

Solution 1 (using a conversion factor):

i. $\text{Volume} = e^3 = (3 \text{ m})^3 = 27 \text{ m}^3$

ii. $27 \text{ m}^3 \times \frac{1 \text{ million cm}^3}{1 \text{ m}^3} = \mathbf{27 \text{ million cm}^3}$

Solution 2 (converting dimensions first):

i. $3 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ m}} = 300 \text{ cm}$

ii. $\text{Volume} = e^3 = (300 \text{ cm})^3 = \mathbf{27 \text{ million cm}^3}$

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

1. What was your reason for choosing which unit to use?

Any unit on the measuring instrument may be used but the decision on what unit to use would depend on how big the object is. In measuring the brick, the glass and the balikbayan box, the appropriate unit to use would be centimeter. In measuring the dimensions of the classroom, the appropriate unit to use would be meter.

2. How did you convert the volume from cc to m^3 or vice versa?

Possible answer would be converting the dimensions to the desired units first before solving for the volume.

3. How did you convert the volume from cc or m^3 to the English units for volume?

Possible answer would be by converting the dimensions into English units first before solving for the volume.

III. Exercises:

Answer the following items. Show your solutions.

1. Convert 10 m^3 to ft^3
2. Convert 12 cups to mL
3. A cylindrical water tank has a diameter of 4 feet and a height of 7 feet while a water tank shaped like a rectangular prism has a length of 1 m, a width of 2 meters and a height of 2 meters. Which of the two tanks can hold more water? By how many cubic meters?

C.

I. Activity:

Problem: The rectangular water tank of a fire truck measures 3 m by 4 m by 5 m. How many liters of water can the fire truck hold?

Volume (Continued)

While capacities of containers are obtained by measuring its dimensions, fluid volume may also be expressed using Metric or English units for fluid volume such as liters or gallons. It is then essential to know how to convert commonly used units for volume into commonly used units for measuring fluid volume.

While the cubic meter is the SI unit for volume, the liter is also widely accepted as a SI-derived unit for capacity. In 1964, after several revisions of its definition, the General Conference on Weights and Measures (CGPM) finally defined a liter as equal to one cubic decimeter. Later, the letter L was also accepted as the symbol for liter.

This conversion factor may also be interpreted in other ways. Check out the conversion factors below:

$$\begin{aligned}1 \text{ L} &= 1 \text{ dm}^3 \\1 \text{ mL} &= 1 \text{ cc} \\1,000 \text{ L} &= 1 \text{ m}^3\end{aligned}$$

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the problem above:

$$\begin{aligned}\text{Step 1: } V &= lwh \\&= 3\text{m} \times 4\text{m} \times 5\text{m} \\&= 60 \text{ m}^3\end{aligned}$$

$$\text{Step 2: } 60 \text{ m}^3 \times \frac{1,000 \text{ L}}{1 \text{ m}^3} = \mathbf{60,000 \text{ L}}$$

III. Exercise:

Instructions: Answer the following items. Show your solution.

1. A spherical fish bowl has a radius of 21 cm. How many mL of water is needed to fill half the bowl?
2. A rectangular container van needs to be filled with identical cubical balikbayan boxes. If the container van's length, width and height are 16 ft, 4 ft and 6ft, respectively, while each balikbayan box has an edge of 2 ft, what is the maximum number of balikbayan boxes that can be placed inside the van?
3. A drinking glass has a height of 4 in, a length of 2 in and a width of 2 in while a baking pan has a width of 4 in, a length of 8 in and a depth of 2 in. If the baking pan is to be filled with water up to half its depth using the drinking glass, how many glasses full of water would be needed?

D.**Activity:**

Instructions: Fill the table below according to the column headings. Choose which of the available instruments is the most appropriate in measuring the given object's weight. For the weight, choose only one of the given units.

	INSTRUMENT*	WEIGHT		
		Gram	Kilogram	Pound
¢25-coin				
₱ 5-coin				
Small toy marble				
Piece of brick				
Yourself				

*Available instruments: triple-beam balance, nutrition (kitchen) scale, bathroom scale
Answer the following questions:

1. What was your reason for choosing which instrument to use?
2. What was your reason for choosing which unit to use?
3. What other kinds of instruments for measuring weight do you know?
4. What other units of weight do you know?

Mass/ Weight

In common language, mass and weight are used interchangeably although weight is the more popular term. Oftentimes in daily life, it is the mass of the given object which is called its weight. However, in the scientific community, mass and weight are two different measurements. Mass refers to the amount of matter an object has while weight is the gravitational force acting on an object.

Weight is often used in daily life, from commerce to food production. The base SI unit for weight is the kilogram (kg) which is almost exactly equal to the mass of one liter of water. For the English System of Measurement, the base unit for weight is the pound (lb). Since both these units are used in Philippine society, knowing how to convert from pound to kilogram or vice versa is important. Some of the more common Metric units are the gram (g) and the milligram (mg) while another commonly used English unit for weight is ounces (oz). Here are some of the conversion factors for these units:

$$\begin{array}{lll}
 1 \text{ kg} = 2.2 \text{ lb} & 1 \text{ g} = 1000 \text{ mg} & 1 \text{ metric ton} = 1000 \text{ kg} \\
 1 \text{ kg} = 1000 \text{ g} & 1 \text{ lb} = 16 \text{ oz} &
 \end{array}$$

Use these conversion factors to convert common weight units to the desired unit. For example:

$$\text{Convert } 190 \text{ lb to kg: } 190 \text{ lb} \times \frac{1 \text{ kg}}{2.2 \text{ lb}} = 86.18 \text{ kg}$$

II. Questions to Ponder (Post-Activity Discussion)

1. What was your reason for choosing which instrument to use?
Possible reasons would include how heavy the object to be weighed to the capacity of the weighing instrument.
2. What was your reason for choosing which unit to use?
The decision on which unit to use would depend on the unit used by the weighing instrument. This decision will also be influenced by how heavy the object is.

3. What other kinds of instruments for measuring weight do you know?
Other weighing instruments include the two-pan balance, the spring scale, the digital scales.
4. What other common units of weight do you know?
Possible answers include ounce, carat and ton.

III. Exercise:

Answer the following items. Show your solution.

1. Complete the table above by converting the measured weight into the specified units.
2. When Sebastian weighed his balikbayan box, its weight was 34 kg. When he got to the airport, he found out that the airline charged \$5 for each lb in excess of the free baggage allowance of 50 lb. How much will Sebastian pay for the excess weight?
3. A forwarding company charges P1,100 for the first 20 kg and P60 for each succeeding 2 kg for freight sent to Europe. How much do you need to pay for a box weighing 88 lb?

Summary

In this lesson, you learned: 1) how to determine the volume of selected regularly-shaped solids; 2) that the base SI unit for volume is the cubic meter; 3) how to convert Metric and English units of volume from one to another; 4) how to solve problems involving volume or capacity; 5) that mass and weight are two different measurements and that what is commonly referred to as weight in daily life is actually the mass; 6) how to use weighing instruments to measure the mass/weight of objects and people; 7) how to convert common Metric and English units of weight from one to another; 8) how to solve problems involving mass / weight.

Lesson 17: Measuring Angles, Time and Temperature

Prerequisite Concepts: Basic concepts of measurement, ratios

About the Lesson:

This lesson should reinforce your prior knowledge and skills on measuring angle, time and temperature as well as meter reading. A good understanding of this concept would not only be useful in your daily lives but would also help you in geometry and physical sciences.

Objectives:

At the end of the lesson, you should be able to:

11. estimate or approximate measures of angle, time and temperature;
12. use appropriate instruments to measure angles, time and temperature;
13. solve problems involving time, speed, temperature and utilities usage (meter reading).

Lesson Proper

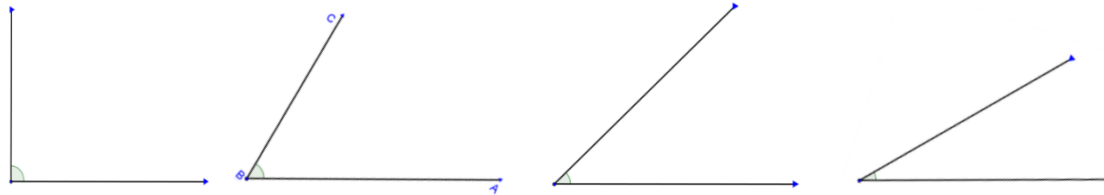
A.

I. Activity:

Material needed:

Protractor

Instruction: Use your protractor to measure the angles given below. Write your answer on the line provided.



1. _____

2. _____

3. _____

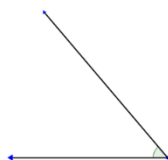
4. _____

Angles

Derived from the Latin word *angulus*, which means corner, an angle is defined as a figure formed when two rays share a common endpoint called the vertex. Angles are measured either in degree or radian measures. A protractor is used to determine the measure of an angle in degrees. In using the protractor, make sure that the cross bar in the middle of the protractor is aligned with the vertex and one of the legs of the angle is aligned with one side of the line passing through the cross bar. The measurement of the angle is determined by its other leg.

Answer the following items:

1. Estimate the measurement of the angle below. Use your protractor to check your estimate.



Estimate _____

Measurement using the protractor _____

2. What difficulties did you meet in using your protractor to measure the angles?

3. What can be done to improve your skill in estimating angle measurements?

II. Questions to Ponder (Post-activity discussion):

1. Estimate the measurement of the angles below. Use your protractor to check your estimates.

Measurement = 50°

2. What difficulties did you meet in using your protractor to measure the angles?

One of the difficulties you may encounter would be on the use of the protractor and the angle orientation. Aligning the cross bar and base line of the protractor with the vertex and an angle leg, respectively, might prove to be confusing at first, especially if the angle opens in the clockwise orientation. Another difficulty arises if the length of the leg is too short such that it won't reach the tick marks on the protractor. This can be remedied by extending the leg.

3. What can be done to improve your skill in estimating angle measurements?

You may familiarize yourself with the measurements of the common angles like the angles in the first activity and use these angles in estimating the measurement of other angles.

III. Exercise:

Instructions: Estimate the measurement of the given angles, then check your estimates by measuring the same angles using your protractor.

ANGLE			
	A	B	C
	ESTIMATE		
MEASUREMENT			

B.

I. Activity

Problem: An airplane bound for Beijing took off from the Ninoy Aquino International Airport at 11:15 a.m. Its estimated time of arrival in Beijing is at 1550 hrs. The distance from Manila to Beijing is 2839 km.

1. What time (in standard time) is the plane supposed to arrive in Beijing?
2. How long is the flight?
3. What is the plane's average speed?

Time and Speed

The concept of time is very basic and is integral in the discussion of other concepts such as speed. Currently, there are two types of notation in stating time, the 12-hr notation (standard time) or the 24-hr notation (military or astronomical time). Standard time makes use of a.m. and p.m. to distinguish between the time

from 12 midnight to 12 noon (a.m. or *ante meridiem*) and from 12 noon to 12 midnight (p.m. or *post meridiem*). This sometimes leads to ambiguity when the suffix of a.m. and p.m. are left out. Military time prevents this ambiguity by using the 24-hour notation where the counting of the time continues all the way to 24. In this notation, 1:00 p.m. is expressed as 1300 hours or 5:30 p.m. is expressed as 1730 hours.

Speed is the rate of an object's change in position along a line. Average speed is determined by dividing the distance travelled by the time spent to cover the distance (Speed = $\frac{\text{distance}}{\text{time}}$ or $S = \frac{d}{t}$, read as "distance per time"). The base SI unit for speed is meters per second ($\frac{\text{m}}{\text{s}}$). The commonly used unit for speed is Kilometers/hour (kph or $\frac{\text{km}}{\text{h}}$) for the Metric system and miles/hour (mph or $\frac{\text{mi}}{\text{hr}}$) for the English system.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

1. What time (in standard time) is the plane supposed to arrive in Beijing?

3:50 p.m.

2. How long is the flight?

1555 hrs – 1115 hrs = 4 hrs, 40 minutes or 4.67 hours

3. What is the plane's average speed?

$$S = d/t$$

$$= 2839 \text{ km} / 4.67 \text{ hrs}$$

$$= 607.92 \text{ kph}$$

III. Exercise:

Answer the following items. Show your solutions.

1. A car left the house and travelled at an average speed of 60 kph. How many minutes will it take for the car to reach the school which is 8 km away from the house?

2. Sebastian stood at the edge of the cliff and shouted facing down. He heard the echo of his voice 4 seconds after he shouted. Given that the speed of sound in air is 340 m / s, how deep is the cliff?

3. Maria ran in a 42-km marathon. She covered the first half of the marathon from 0600 hrs to 0715 hours and stopped to rest. She resumed running and was able to cover the remaining distance from 0720 hrs to 0935 hrs. What was Maria's average speed for the entire marathon?

C.

I. Activity:

Problem: Zale, a Cebu resident, was packing his suitcase for his trip to New York City the next day for a 2-week vacation. He googled New York weather and found out the average temperature there is 59°F. Should he bring a sweater? What data should Zale consider before making a decision?

Temperature

Temperature is the measurement of the degree of hotness or coldness of an object or substance. While the commonly used units are Celsius (°C) for the Metric system and Fahrenheit (°F) for the English system, the base SI unit for temperature is the Kelvin (K). Unlike the Celsius and Fahrenheit which are considered degrees,

the Kelvin is considered as an absolute unit of measure and therefore can be worked on algebraically.

Hereunder are some conversion factors:

$$^{\circ}\text{C} = \left(\frac{5}{9}\right)(^{\circ}\text{F} - 32)$$

$$^{\circ}\text{F} = \left(\frac{9}{5}\right)(^{\circ}\text{C}) + 32$$

$$\text{K} = ^{\circ}\text{C} + 273.15$$

For example:

$$\begin{aligned}\text{Convert } 100^{\circ}\text{C to } ^{\circ}\text{F: } ^{\circ}\text{F} &= \left(\frac{9}{5}\right)(100^{\circ}\text{C}) + 32 \\ &= 180 + 32 \\ &= 212^{\circ}\text{F}\end{aligned}$$

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the problem above:

1. What data should Zale consider before making a decision?

In order to determine whether he should bring a sweater or not, Zale needs to compare the average temperature in NYC to the temperature he is used to which is the average temperature in Cebu. He should also express both the average temperature in NYC and in Cebu in the same units for comparison.

2. Should Zale bring a sweater?

The average temperature in Cebu is between 24 – 32 °C. Since the average temperature in NYC is 59°F which is equivalent to 15°C, Zale should probably bring a sweater since the NYC temperature is way below the temperature he is used to. Better yet, he should bring a jacket just to be safe.

III. Exercise:

Instructions: Answer the following items. Show your solution.

1. Convert 14°F to K.
2. Maria was preparing the oven to bake brownies. The recipe's direction was to pre-heat the oven to 350°F but her oven thermometer was in °C. What should be the thermometer reading before Maria puts the baking pan full of the brownie mix in the oven?

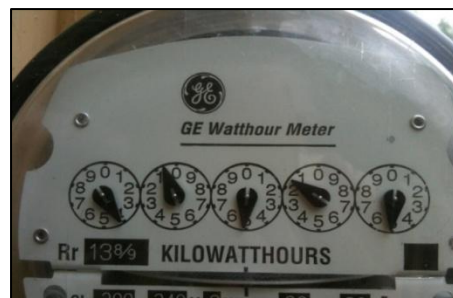
D.

Activity:

Instructions: Use the pictures below to answer the questions that follow.



Initial electric meter reading at 0812 hrs
on 14 Feb 2012



Final electric meter reading at 0812 hrs
on 15 Feb 2012

1. What was the initial meter reading? Final meter reading?
2. How much electricity was consumed during the given period?

3. How much will the electric bill be for the given time period if the electricity charge is P9.50 / kiloWatt-hour?

Reading Your Electric Meter

Nowadays, reading the electric meter would be easier considering that the newly-installed meters are digital but most of the installed meters are still dial-based. Here are the steps in reading the electric meter:

- a. To read your dial-based electric meter, read the dials from left to right.
- b. If the dial hand is between numbers, the smaller of the two numbers should be used. If the dial hand is on the number, check out the dial to the right. If the dial hand has passed zero, use the number at which the dial hand is pointing. If the dial hand has not passed zero, use the smaller number than the number at which the dial hand is pointing.
- c. To determine the electric consumption for a given period, subtract the initial reading from the final reading.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions above:

1. What was the initial meter reading? final meter reading?

The initial reading is 40493 kWh. For the first dial from the left, the dial hand is on the number 4 so you look at the dial immediately to the right which is the second dial. Since the dial hand of the second dial is past zero already, then the reading for the first dial is 4. For the second dial, since the dial hand is between 0 and 1 then the reading for the second dial is 0. For the third dial from the left, the dial hand is on the number 5 so you look at the dial immediately to the right which is the fourth dial. Since the dial hand of the fourth dial has not yet passed zero, then the reading for the third dial is 4. The final reading is 40515 kWh.

2. How much electricity was consumed during the given period?

Final reading – initial reading = 40515 kWh – 40493 kWh = 22 kWh

3. How much will the electric bill be for the given time period if the electricity charge is ₱ 9.50 / kiloWatt-hour?

*Electric bill = total consumption x electricity charge
= 22 kWh x P9.50 / kWh
= P209*

III. Exercise:

Answer the following items. Show your solution.

1. The pictures below show the water meter reading of Sebastian’s house.



Initial meter reading at 0726 hrs on 20 February 2012



Final meter reading at 0725 hrs on 21 February 2012

If the water company charges P14 / cubic meter of water used, how much must Sebastian pay the water company for the given period?

2. The pictures below show the electric meter reading of Maria's canteen.



Initial meter reading at 1600 hrs on 20 Feb 2012



Final meter reading @ 1100 hrs on 22 Feb 2012

If the electric charge is P9.50 / kWh, how much will Maria pay the electric company for the given period?

3. The pictures below show the electric meter reading of a school.



Initial meter reading @ 1700 hrs on 15 July 2012



Final meter reading @ 1200 hrs on 16 July 2012

Assuming that the school's average consumption remains the same until 1700 hrs of 15 August 2012 and the electricity charge is P9.50 / kWh, how much will the school be paying the electric company?

Summary

In this lesson, you learned:

1. how to measure angles using a protractor;
2. how to estimate angle measurement;
3. express time in 12-hr or 24-hr notation;
4. how to measure the average speed as the quotient of distance over time;
5. convert units of temperature from one to the other;
6. solve problems involving time, speed and temperature;
7. read utilities usage.

Lesson 18: Constants, Variables and Algebraic Expressions

Prerequisite Concepts: Real Number Properties and Operations

About the Lesson:

This lesson is an introduction to the concept of constants, unknowns and variables and algebraic expressions. Familiarity with this concept is necessary in laying a good foundation for Algebra and in understanding and translating mathematical phrases and sentences, solving equations and algebraic word problems as well as in grasping the concept of functions.

Objectives:

At the end of the lesson, you should be able to:

1. Differentiate between constants and variables in a given algebraic expression
2. Evaluate algebraic expressions for given values of the variables

Lesson Proper

I. Activity

A. Instructions: Complete the table below according to the pattern you see.

ROW	1 ST TERM	2 ND TERM
a.	1	5
b.	2	6
c.	3	7
d.	4	
e.	5	
f.	6	
g.	59	
h.	Any number n	

B. Using Table A as your basis, answer the following questions:

1. What did you do to determine the 2nd term for rows d to f?
2. What did you do to determine the 2nd term for row g?
3. How did you come up with your answer in row h?
4. What is the relation between the 1st and 2nd terms?
5. Express the relation of the 1st and 2nd terms in a mathematical sentence.

II. Questions to Ponder (Post-Activity Discussion)

A. The 2nd terms for rows d to f are 8, 9 and 10, respectively. The 2nd term in row g is 63. The 2nd term in row h is the sum of a given number n and 4.

B.

1. One way of determining the 2nd terms for rows d to f is to add 1 to the 2nd term of the preceding row (e.g. $7 + 1 = 8$). Another way to determine the 2nd term would be to add 4 to its corresponding 1st term (e.g. $4 + 4 = 8$).
2. Since from row f, the first term is 6, and from 6 you add 53 to get 59, to get the 2nd term of row g, $10 + 53 = 63$. Of course, you could have simply added 4 to 59.

3. The answer in row h is determined by adding 4 to n , which represents any number.
4. The 2nd term is the sum of the 1st term and 4.
5. To answer this item better, we need to be introduced to Algebra first.

Algebra

We need to learn a new language to answer item 5. The name of this language is Algebra. You must have heard about it. However, Algebra is not entirely a new language to you. In fact, you have been using its applications and some of the terms used for a long time already. You just need to see it from a different perspective.

Algebra comes from the Arabic word, *al-jabr* (which means restoration), which in turn was part of the title of a mathematical book written around the 820 AD by Arab mathematician, *Muhammad ibn Musa al-Khwarizmi*. While this book is widely considered to have laid the foundation of modern Algebra, history shows that ancient Babylonian, Greek, Chinese and Indian mathematicians were discussing and using algebra a long time before this book was published.

Once you've learned this new language, you'll begin to appreciate how powerful it is and how its applications have drastically improved our way of life.

III. Activity

Instructions: How do you understand the following symbols and expressions?

SYMBOLS / EXPRESSIONS	MEANING
1. x	
2. $2 + 3$	
3. $=$	

IV. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the previous activity:

1. You might have thought of x as the multiplication sign. From here on, x will be considered a symbol that stands for any value or number.
2. You probably thought of $2 + 3$ as equal to 5 and must have written the number 5. Another way to think of $2 + 3$ is to read it as the sum of 2 and 3.
3. You must have thought, "Alright, what am I supposed to compute?" The sign " $=$ " may be called the equal sign by most people but may be interpreted as a command to carry out an operation or operations. However, the equal sign is also a symbol for the relation between the expressions on its left and right sides, much like the less than " $<$ " and greater than " $>$ " signs.

The Language Of Algebra

The following are important terms to remember.

- a. **constant** – a constant is a number on its own. For example, 1 or 127;
- b. **variable** – a variable is a symbol, usually letters, which represent a value or a number. For example, a or x . In truth, you have been dealing with variables since pre-school in the form of squares (\square), blank lines ($\underline{\quad}$) or other symbols used to represent the unknowns in some mathematical sentences or phrases;

- c. **term** – a term is a constant or a variable or constants and variables multiplied together. For example, 4, xy or $8yz$. The term's number part is called the **numerical coefficient** while the variable or variables is/are called the **literal coefficient/s**. For the term $8yz$, the **numerical coefficient** is 8 and the **literal coefficients** are yz ,
- d. **expression** – an Algebraic expression is a group of terms separated by the plus or minus sign. For example, $x - 2$ or $4x + \frac{1}{2}y - 45$

Problem: Which of the following is/are equal to 5?

- a. $2 + 3$ b. $6 - 1$ c. $\frac{10}{2}$ d. $1+4$ e. all of these

Discussion: The answer is e since $2 + 3$, $6 - 1$, $\frac{10}{2}$ and $1 + 4$ are all equal to 5.

Notation

Since the letter x is now used as a variable in Algebra, it would not only be funny but confusing as well to still use x as a multiplication symbol. Imagine writing the product of 4 and a value x as $4xx$! Thus, Algebra simplifies multiplication of constants and variables by just writing them down beside each other or by separating them using only parentheses or the symbol “ \bullet ”. For example, the product of 4 and the value x (often read as four x) may be expressed as $4x$, $4(x)$ or $4\bullet x$. Furthermore, division is more often expressed in fraction form. The division sign \div is now seldom used.

Problem: Which of the following equations is true?

- a. $12 + 5 = 17$
 b. $8 + 9 = 12 + 5$
 c. $6 + 11 = 3(4 + 1) + 2$

Discussion: All of the equations are true. In each of the equations, both sides of the equal sign give the same number though expressed in different forms. In a) 17 is the same as the sum of 12 and 5. In b) the sum of 8 and 9 is 17 thus it is equal to the sum of 12 and 5. In c) the sum of 6 and 11 is equal to the sum of 2 and the product of 3 and the sum of 4 and 1.

On Letters and Variables

Problem: Let x be any real number. Find the value of the expression $3x$ (the product of 3 and x , remember?) if

- a) $x = 5$ b) $x = \frac{1}{2}$ c) $x = -0.25$

Discussion: The expression $3x$ means multiply 3 by any real number x . Therefore,
 a) If $x = 5$, then $3x = 3(5) = 15$.
 b) If $x = \frac{1}{2}$, then $3x = 3(\frac{1}{2}) = \frac{3}{2}$
 c) If $x = -0.25$, then $3x = 3(-0.25) = -0.75$

The letters such as x , y , n , etc. do not always have specific values assigned to them. When that is the case, simply think of each of them as any number. Thus, they can be added ($x + y$), subtracted ($x - y$), multiplied (xy), and divided ($\frac{x}{y}$) like any real number.

Problem: Recall the formula for finding the perimeter of a rectangle, $P = 2L + 2W$. This means you take the sum of twice the length and twice the width of the rectangle to get the perimeter. Suppose the length of a rectangle is 6.2 cm and the width is $\frac{1}{8}$ cm. What is the perimeter?

Discussion: Let $L = 6.2$ cm and $W = \frac{1}{8}$ cm. Then,
 $P = 2(6.2) + 2(\frac{1}{8}) = 12.4 + \frac{1}{4} = 12.65$ cm

V. Exercises:

- Which of the following is considered a constant?
 - f
 - \square
 - 500
 - $42x$
- Which of the following is a term?
 - $23m + 5$
 - $(2)(6x)$
 - $x - y + 2$
 - $\frac{1}{2}x - y$
- Which of the following is equal to the product of 27 and 2?
 - 29
 - $49 + 6$
 - $60 - 6$
 - $11(5)$
- Which of the following makes the sentence $69 - 3 = \underline{\quad} + 2$ true?
 - 33
 - 64
 - 66
 - 68
- Let $y = 2x + 9$. What is y when $x = 5$?
 - 118
 - 34
 - 28
 - 19

Let us now answer item B.5. of the initial problem using Algebra:

- The relation of the 1st and 2nd terms of Table A is “the 2nd term is the sum of the 1st term and 4”. To express this using an algebraic expression, we use the letters n and y as the variables to represent the 1st and 2nd terms, respectively. Thus, if n represents the 1st term and y represents the 2nd term, then

$$y = n + 4.$$

FINAL PROBLEM:

A. Fill the table below:

TABLE B		
ROW	1 ST TERM	2 ND TERM
a.	10	23
b.	11	25
c.	12	27
d.	13	
e.	15	
f.	18	
g.	37	
h.	n	

B. Using Table B as your basis, answer the following questions:

1. What did you do to determine the 2nd term for rows d to f?
2. What did you do to determine the 2nd term for row g?
3. How did you come up with your answer in row h?
4. What is the relation between the 1st and 2nd terms?
5. Express the relation of the 1st and 2nd terms using an algebraic expression.

Summary

In this lesson, you learned about constants, letters and variables, and algebraic expressions. You learned that the equal sign means more than getting an answer to an operation; it just means that expressions on either side have equal values. You also learned how to evaluate algebraic expressions when values are assigned to letters.

Lesson 19: Verbal Phrases and Mathematical Phrases

Time: 2 hours

Prerequisite Concepts: Real Numbers and Operations on Real Numbers

About the Lesson:

This lesson is about verbal phrases and sentences and their equivalent expressions in mathematics. This lesson will show that mathematical or algebraic expressions are also meaningful.

Objectives

In this lesson, you will be able to translate verbal phrases to mathematical phrases and vice versa.

Lesson Proper

I. Activity 1

Directions: Match each verbal phrase under Column A to its mathematical phrase under Column B. Each number corresponds to a letter which will reveal a quotation if answered correctly. A letter may be used more than once.

Column A		Column B	
_____ 1.	The sum of a number and three	A.	$x + 3$
_____ 2.	Four times a certain number decreased by one	B.	$3 + 4x$
_____ 3.	One subtracted from four times a number	E.	$4 + x$
_____ 4.	A certain number decreased by two	I.	$x + 4$
_____ 5.	Four increased by a certain number	L.	$4x - 1$
_____ 6.	A certain number decreased by three	M.	$x - 2$
_____ 7.	Three more than a number	N.	$x - 3$
_____ 8.	Twice a number decreased by three	P.	$3 - x$
_____ 9.	A number added to four	Q.	$2 - x$
_____ 10.	The sum of four and a number	R.	$2x - 3$
_____ 11.	The difference of two and a number	U.	$4x + 3$
_____ 12.	The sum of four times a number and three		
_____ 13.	A number increased by three		
_____ 14.	The difference of four times a number and one		

II. Question to Ponder (Post-Activity Discussion)

Which phrase was easy to translate? _____

Translate the mathematical expression $2(x-3)$ in at least two ways.

Did you get the quote, “ALL MEN ARE EQUAL”? If not, what was your mistake?

III. Activity 2

Directions: Choose the words or expressions inside the boxes and write it under its respective symbol.

plus increased by	more than subtracted from	times multiplied by	divided by ratio of	is less than is greater than or equal to
is greater than	the quotient of	of	is at most	is less than or equal to
the sum of is at least	the difference of the product of	diminished by decreased by	less than is not equal to	added to minus

+	–	x	÷	<	>	≤	≥	≠
increased by	decreased by	multiplied by	ratio of	is less than	is greater than	is less than or equal to	is greater than or equal to	is not equal to
added to	subtracted from	of	the quotient of			is at most	is at least	
the sum of	the difference of	the product of						
more than	less than							
	diminished by							

IV. Question to Ponder (Post-Activity Discussion)

1. Addition would indicate an increase, a putting together, or combining. Thus, phrases like *increased by* and *added to* are addition phrases.
2. Subtraction would indicate a lessening, diminishing action. Thus, phrases like *decreased by*, *less*, *diminished by* are subtraction phrases.
3. Multiplication would indicate a multiplying action. Phrases like *multiplied by* or *n times* are multiplication phrases.
4. Division would indicate partitioning, a quotient, and a ratio. Phrases such as *divided by*, *ratio of*, and *quotient of* are common for division.
5. The inequalities are indicated by phrases such as *less than*, *greater than*, *at least*, and *at most*.
6. Equalities are indicated by phrases like *the same as* and *equal to*.

V. THE TRANSLATION OF THE “=” SIGN

Directions: The table below shows two columns, A and B. Column A contains mathematical sentences while Column B contains their verbal translations. Observe the items under each column and compare. Answer the proceeding questions.

Column A Mathematical Sentence	Column B Verbal Sentence
$x + 5 = 4$	The sum of a number and 5 is 4.
$2x - 1 = 1$	Twice a number decreased by 1 is equal to 1.
$7 + x = 2x + 3$	Seven added by a number x is equal to twice the same number increased by 3.
$3x = 15$	Thrice a number x yields 15.
$x - 2 = 3$	Two less than a number x results to 3.

VI. Question to Ponder (Post-Activity Discussion)

- 1) Based on the table, what do you observe are the common verbal translations of the “=” sign? “is”, “is equal to”
- 2) Can you think of other verbal translations for the “=” sign? “results in”, “becomes”
- 3) Use the phrase “is equal to” on your own sentence.
- 4) Write your own pair mathematical sentence and its verbal translation on the last row of the table.

VII. Exercises:

A. Directions: Write your responses on the space provided.

1. Write the verbal translation of the formula for converting temperature from

Celsius (C) to Fahrenheit (F) which is $F = \frac{9}{5}C + 32$.

2. Write the verbal translation of the formula for converting temperature from

Fahrenheit (F) to Celsius (C) which is $C = \frac{5}{9}F - 32$.

3. Write the verbal translation of the formula for simple interest: $I = PRT$, where I is simple interest, P is Principal Amount, R is Rate and T is time in years.
-

4. The perimeter (P) of a rectangle is twice the sum of the length (L) and width (W). Express the formula of the perimeter of a rectangle in algebraic expressions using the indicated variables.
-

5. The area (A) of a rectangle is the product of length (L) and width (W).

6. The perimeter (P) of a square is four times its side (S).

7. Write the verbal translation of the formula for Area of a Square (A): $A = s^2$, where s is the length of a side of a square.

8. The circumference (C) of a circle is twice the product of π and radius (r).

9. Write the verbal translation of the formula for Area of a Circle (A): $A = \pi r^2$, where r is the radius.

10. The midline (k) of a trapezoid is half the sum of the bases (a and b) or the sum of the bases (a and b) divided by 2.

11. The area (A) of a trapezoid is half the product of the sum of the bases (a and b) and height (h).

12. The area (A) of a triangle is half the product of the base (b) and height (h).

13. The sum of the angles of a triangle (A, B and C) is 180° .

14. Write the verbal translation of the formula for Area of a Rhombus (A): $A = \frac{1}{2} d_1 d_2$, where d_1 and d_2 are the lengths of diagonals.

15. Write the verbal translation of the formula for the Volume of a rectangular parallelepiped (V): $A = lwh$, where l is the length, w is the width and h is the height.

16. Write the verbal translation of the formula for the Volume of a sphere (V): $V = \frac{4}{3}\pi r^3$, where r is the radius.

17. Write the verbal translation of the formula for the Volume of a cylinder (V): $V = \pi r^2 h$, where r is the radius and h is the height.

18. The volume of the cube (V) is the cube of the length of its edge (a). Or the volume of the cube (V) is the length of its edge (a) raised to 3. Write its formula.

B. Directions: Write as many verbal translations as you can for this mathematical sentence.

$$3x - 2 = -4$$

C. REBUS PUZZLE

Try to answer this puzzle!

What number must replace the letter x?

$$x + (\text{bone}) - (\text{b}) = (\text{tiger}) - (\text{kit})$$

SUMMARY

In this lesson, you learned that verbal phrases can be written in both words and in mathematical expressions. You learned common phrases associated with addition, subtraction, multiplication, division, the inequalities and the equality. With this lesson, you must realize by now that mathematical expressions are also meaningful.

GRADE 7 MATH LEARNING GUIDE

Lesson 20: Polynomials

Time: 1.5 hours

Pre-requisite Concepts: Constants, Variables, Algebraic expressions

About the Lesson: This lesson introduces to students the terms associated with polynomials. It discusses what polynomials are.

Objectives:

In this lesson, the students must be able to:

- 1) Give examples of polynomials, monomials, binomials, and trinomials;
- 2) Identify the base, coefficient, terms and exponent sin a given polynomial.

Lesson Proper:

I. A. Activity 1: Word Hunt

Find the following words inside the box.

- BASE
- COEFFICIENT
- DEGREE
- EXPONENT
- TERM
- CONSTANT
- BINOMIAL
- MONOMIAL
- POLYNOMIAL
- TRINOMIAL
- CUBIC
- LINEAR
- QUADRATIC
- QUINTIC
- QUARTIC

P	C	I	T	N	I	U	Q	Y	N	E	T
P	M	E	X	P	O	N	E	N	T	S	C
C	O	E	F	F	I	C	I	E	N	T	O
Q	N	L	I	N	E	A	R	B	D	R	N
U	O	C	Y	A	P	M	R	A	E	I	S
A	M	R	I	N	L	M	T	S	G	N	T
D	I	U	N	B	O	Q	U	N	R	O	A
R	A	E	Q	P	U	M	V	T	E	M	N
A	L	S	O	B	D	C	I	R	E	I	T
T	A	A	C	U	B	I	N	A	S	A	A
I	U	B	I	N	O	M	I	A	L	L	C
C	I	T	R	A	U	Q	R	T	I	C	B

Definition of Terms

In the algebraic expression $3x^2 - x + 5$, $3x^2$, $-x$ and 5 are called the terms.

Term is a constant, a variable or a product of constant and variable.

In the term $3x^2$, 3 is called the numerical coefficient and x^2 is called the **literal coefficient**.

In the term $-x$ has a **numerical coefficient** which is -1 and a literal coefficient which is x .

The term 5 is called the **constant**, which is usually referred to as the term without a variable.

Numerical coefficient is the constant/number.

Literal coefficient is the variable including its exponent.

The word **Coefficient** alone is referred to as the numerical coefficient.

In the literal coefficient x^2 , x is called the **base** and 2 is called the **exponent**.

Degree is the highest exponent or the highest sum of exponents of the variables in a term.

In $3x^2 - x + 5$, the degree is 2.

In $3x^2y^3 - x^4y^3$ the degree is 7.

Similar Terms are terms having the same literal coefficients.

$3x^2$ and $-5x^2$ are similar because their literal coefficients are the same.

$5x$ and $5x^2$ are NOT similar because their literal coefficients are NOT the same.

$2x^3y^2$ and $-4x^2y^3$ are NOT similar because their literal coefficients are NOT the same.

A **polynomial** is a kind of algebraic expression where each term is a constant, a variable or a product of a constant and variable in which the variable has a whole number (non-negative number) exponent. A polynomial can be a monomial, binomial, trinomial or a multinomial.

An algebraic expression is **NOT** a polynomial if

- 1) the exponent of the variable is NOT a whole number {0, 1, 2, 3..}.
- 2) the variable is inside the radical sign.
- 3) the variable is in the denominator.

Kinds of Polynomial according to the number of terms

- 1) Monomial – is a polynomial with only one term
- 2) Binomial – is polynomial with two terms
- 3) Trinomial – is a polynomial with three terms
- 4) Polynomial – is a polynomial with four or more terms

B. Activity 2

Tell whether the given expression is a polynomial or not. If it is a polynomial, determine its degree and tell its kind according to the number of terms. If it is NOT, explain why.

1) $3x^2$

2) $x^2 - 5xy$

3) 10

4) $3x^2 - 5xy + x^3 + 5$

6) $x^{1/2} - 3x + 4$

7) $\sqrt{2}x^4 - x^7 + 3$

8) $3x^2\sqrt{2x} - 1$

9) $\frac{1}{3}x - \frac{3x^3}{4} + 6$

5) $x^3 - 5x^2 + 3$

10) $\frac{3}{x^2} - x^2 - 1$

Kinds of Polynomial according to its degree

- 1) Constant – a polynomial of degree zero
- 2) Linear – a polynomial of degree one
- 3) Quadratic – a polynomial of degree two
- 4) Cubic – a polynomial of degree three
- 5) Quartic – a polynomial of degree four
- 6) Quintic – a polynomial of degree five

* The next degrees have no universal name yet so they are just called “polynomial of degree ____.”

A polynomial is in **Standard Form** if its terms are arranged from the term with the highest degree, up to the term with the lowest degree.

If the polynomial is in standard form the first term is called the **Leading Term**, the numerical coefficient of the leading term is called the **Leading Coefficient** and the exponent or the sum of the exponents of the variable in the leading term the **Degree** of the polynomial.

The standard form of $2x^2 - 5x^5 - 2x^3 + 3x - 10$ is **$-5x^5 - 2x^3 + 2x^2 + 3x - 10$** .

The terms $-5x^5$ is the leading term, -5 is its leading coefficient and 5 is its degree.

It is a quintic polynomial because its degree is 5 .

C. Activity 3

Complete the table.

Given	Leading Term	Leading Coefficient	Degree	Kind of Polynomial according to the no. of terms	Kind of Polynomial According to the degree	Standard Form
1) $2x + 7$						
2) $3 - 4x + 7x^2$						
3) 10						
4) $x^4 - 5x^3 + 2x - x^2 - 1$						
5) $5x^5 + 3x^3 - x$						
6) $3 - 8x$						
7) $x^2 - 9$						
8) $13 - 2x + x^5$						
9) $100x^3$						

10) $2x^3 - 4x^2$ $+ x^4 - 6$						
----------------------------------	--	--	--	--	--	--

Summary

In this lesson, you learned about the terminologies in polynomials: term, coefficient, degree, similar terms, polynomial, standard form, leading term, leading coefficient.

Lesson 21: Laws of Exponents

Time: 1.5 hours

Pre-requisite Concepts: Multiplication of real numbers

About the Lesson: This lesson is all about the laws of exponents.

Objectives:

In this lesson, the students must be able to:

- 1) define and interpret the meaning of a^n where n is a positive integer;
- 2) derive inductively the Laws of Exponents (restricted to positive integers)
- 3) illustrate the Laws of Exponents.

Lesson Proper

I. Activity 1

Give the product of each of the following as fast as you can.

- 1) $3 \times 3 =$ _____
- 2) $4 \times 4 \times 4 =$ _____
- 3) $5 \times 5 \times 5 =$ _____
- 4) $2 \times 2 \times 2 =$ _____
- 5) $2 \times 2 \times 2 \times 2 =$ _____
- 6) $2 \times 2 \times 2 \times 2 \times 2 =$ _____

II. Development of the Lesson

Discovering the Laws of Exponents

A) $a^n = a \times a \times a \times a \dots$ (n times)

In a^n , a is called the base and n is called the exponent

Exercises

- 1) Which of the following is/are correct?
 - a) $4^2 = 4 \times 4 = 16$
 - b) $2^4 = 2 \times 2 \times 2 \times 2 = 8$
 - c) $2^5 = 2 \times 5 = 10$
 - d) $3^3 = 3 \times 3 \times 3 = 27$
- 2) Give the value of each of the following as fast as you can.
 - a) 2^3
 - b) 2^5
 - c) 3^4
 - d) 10^6

Activity 2

Evaluate the following. Investigate the result. Make a simple conjecture on it. The first two are done for you.

- 1) $(2^3)^2 = 2^3 \cdot 2^3 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$
- 2) $(x^4)^3 = x^4 \cdot x^4 \cdot x^4 = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^{12}$
- 3) $(3^2)^2 =$
- 4) $(2^2)^3 =$

$$5) (a^2)^5 =$$

Did you notice something?

What can you conclude about $(a^n)^m$? What will you do with a , n and m ?

What about these?

$$1) (x^{100})^3$$

$$2) (y^{12})^5$$

Activity 3

Evaluate the following. Notice that the bases are the same. The first example is done for you.

$$1) (2^3)(2^2) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$$

$$2) (x^5)(x^4) =$$

$$3) (3^2)(3^4) =$$

$$4) (2^4)(2^5) =$$

$$5) (x^3)(x^4) =$$

Did you notice something?

What can you conclude about $a^n \cdot a^m$? What will you do with a , n and m ?

What about these?

$$1) (x^{32})(x^{25})$$

$$2) (y^{59})(y^{51})$$

Activity 4

Evaluate each of the following. Notice that the bases are the same. The first example is done for you.

$$1) \frac{2^7}{2^3} = \frac{128}{8} = 16 \quad \text{---}\rightarrow \text{remember that 16 is the same as } 2^4$$

$$2) \frac{3^5}{3^3} =$$

$$3) \frac{4^3}{4^2} =$$

$$4) \frac{2^8}{2^6} =$$

Did you notice something?

What can you conclude about $\frac{a^n}{a^m}$? What will you do with a , n and m ?

What about these?

$$1) \frac{x^{20}}{x^{13}}$$

$$2) \frac{y^{105}}{y^{87}}$$

Summary:

Laws of exponent

1) $a^n = a \cdot a \cdot a \cdot a \cdot a \dots$ (n times)

2) $(a^n)^m = a^{nm}$ *power of powers*

3) $a^n \cdot a^m = a^{m+n}$ *product of a power*

4) $\frac{a^n}{a^m} = a^{n-m}$ *quotient of a power*

5) $a^0 = 1$ where $a \neq 0$ ***law for zero exponent***

What about these?

a) $(7,654,321)^0$

b) $3^0 + x^0 + (3y)^0$

Exercise:

Choose a Law of Exponent to apply. Complete the table and observe. Make a conjecture.

No.	Result	Applying a law of Exponent	GIVEN (Start here)	ANSWER	REASON
1)	←		← $\frac{5}{5}$	→	
2)	←		← $\frac{100}{100}$	→	
3)	←		← $\frac{x}{x}$	→	
4)	←		← $\frac{a^5}{a^5}$	→	

6) a^{-n} and $\frac{1}{a^n}$ ***law for negative exponent***

Can you rewrite the fractions below using exponents and simplify them?

a) $\frac{2}{4}$

b) $\frac{4}{32}$

c) $\frac{27}{81}$

What did you notice?

What about these?

d) x^{-2}

e) 3^{-3}

f) $(5-3)^{-2}$

III. Exercises

A. Evaluate each of the following.

1) 2^8

2) 8^2

3) 5^{-1}

4) 3^{-2}

5) 18^0

6) $(2^3)^3$

7) $(2^4)(2^3)$

8) $(3^2)(2^3)$

9) $x^0 + 3^{-1} - 2^2$

10) $[2^2 - 3^3 + 4^4]^0$

B. Simplify each of the following.

1) $(x^{10})(x^{12})$

2) $(y^{-3})(y^8)$

3) $(m^{15})^3$

4) $(d^{-3})^2$

5) $(a^{-4})^{-4}$

6) $\frac{z^{23}}{z^{15}}$

7) $\frac{b^8}{b^{12}}$

8) $\frac{c^3}{c^{-2}}$

9) $\frac{x^7 y^{10}}{x^3 y^5}$

10) $\frac{a^8 b^2 c^0}{a^5 b^5}$

11) $\frac{a^8 a^3 b^{-2}}{a^{-1} b^{-5}}$

Summary

In these lessons, you learned some laws of exponents.

Lesson 22: Addition and Subtraction of Polynomials

Time: 2 hours

Pre-requisite Concepts: Similar Terms, Addition and Subtraction of Integers

About the Lesson: This lesson will teach students how to add and subtract polynomials using tiles at first and then by paper and pencil after.

Objectives:

In this lesson, the students are expected to:

- 1) add and subtract polynomials;
- 2) solve problems involving polynomials.

Lesson Proper:

I. Activity 1

Familiarize yourself with the Tiles below:



Stands for $(+1)$



Stands for $(+x)$



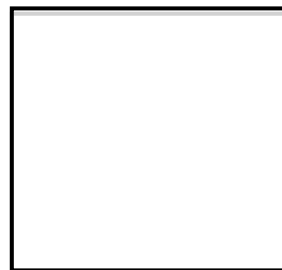
Stands for (-1)



Stands for $(-x)$



Stands for $(+x^2)$



Stands for $(-x^2)$

Can you represent the following quantities using the above tiles?

1. $x - 2$
2. $4x + 1$

Activity 2.

Use the tiles to find the sum of the following polynomials;

1. $5x + 3x$
2. $(3x - 4) - 6x$
3. $(2x^2 - 5x + 2) + (3x^2 + 2x)$

Can you come up with the rules for adding polynomials?

II. Questions/Points to Ponder (Post-Activity Discussion)

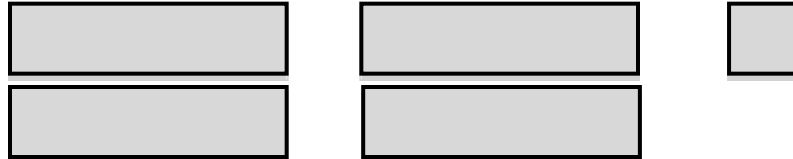
The tiles can make operations on polynomials easy to understand and do.

Let us discuss the first activity.

1. To represent $x - 2$, we get one (+x) tile and two (-1) tiles.



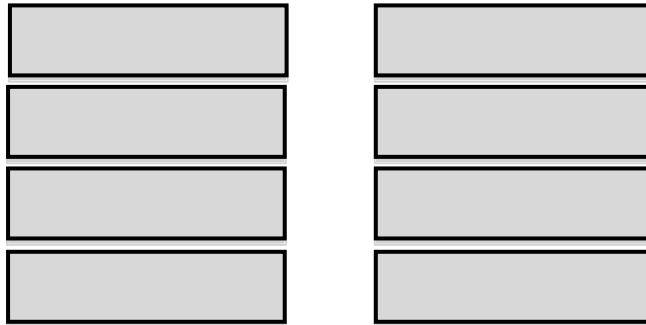
2. To represent $4x + 1$, we get four (+x) tiles and one (+1) tile.



What about the second activity? Did you pick out the correct tiles?

1. $5x + 3x$

Get five (+x tiles) and three more (+x) tiles. How many do you have in all?

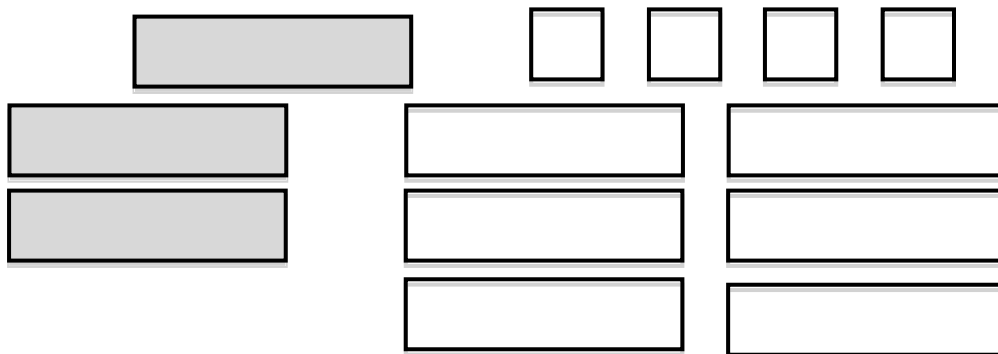


There are eight (+x) altogether. Therefore, $5x + 3x = 8x$.

2. $(3x - 4) - 6x$

Get three (+x) tiles and four (-1) tiles to represent $(3x - 4)$. Add six (-x) tiles.

[Recall that subtraction also means adding the negative of the quantity.]



Now, recall further that a pair of one (+x) and one (-x) is zero. What tiles do you have left?

That's right, if you have with you three (-x) and four (-1), then you are correct. That means the sum is (-3x -4).

$$3. \quad (2x^2 - 5x + 2) + (3x^2 + 2x)$$

What tiles would you put together? You should have two (+x²), five (-x) and two (+1) tiles then add three (+x²) and two (+x) tiles. Matching the pairs that make zero, you have in the end five (+x²), three (-x), and two (+1) tiles. The sum is 5x² - 3x + 2.

Or, using your pen and paper,

$$(2x^2 - 5x + 2) + (3x^2 + 2x) = (2x^2 + 3x^2) + (-5x + 2x) + 2 = 5x^2 - 3x + 2$$

Rules for Adding Polynomials

To add polynomials, simply combine similar terms. To combine similar terms, get the sum of the numerical coefficients and annex the same literal coefficients. If there is more than one term, for convenience, write similar terms in the same column.

Do you think you can add polynomials now without the tiles?

Perform the operation.

1) Add $4a - 3b + 2c$, $5a + 8b - 10c$ and $-12a + c$.

$$\begin{array}{r} 4a - 3b + 2c \\ 5a + 8b - 10c \\ + -12a \quad \quad + c \\ \hline \end{array}$$

2) Add $13x^4 - 20x^3 + 5x - 10$ and $-10x^2 - 8x^4 - 15x + 10$.

$$\begin{array}{r} 13x^4 - 20x^3 \quad \quad + 5x - 10 \\ + -8x^4 \quad \quad - 10x^2 - 15x + 10 \\ \hline \end{array}$$

Rules for Subtracting Polynomials

To subtract polynomials, change the sign of the subtrahend then proceed to the addition rule. Also, remember what subtraction means. It is adding the negative of the quantity.

Perform the operation.

1) $5x - 13x = 5x + (-5x) + (-8x) = -8x$

$$\begin{array}{r} 2) \quad 2x^2 - 15x + 25 \\ \quad - 3x^2 + 12x - 18 \\ \hline \end{array} \qquad \begin{array}{r} 2x^2 - 15x + 25 \\ + -3x^2 - 12x + 18 \\ \hline \end{array}$$

3) $(30x^3 - 50x^2 + 20x - 80) - (17x^3 + 26x + 19)$

$$\begin{array}{r} 30x^3 - 50x^2 + 20x - 80 \\ + -17x^3 \quad \quad - 26x - 19 \\ \hline \end{array}$$

III. Exercises

A. Perform the indicated operation, first using the tiles when applicable, then using paper and pen.

1) $3x + 10x$

6) $10xy - 8xy$

2) $12y - 18y$

7) $20x^2y^2 + 30x^2y^2$

3) $14x^3 + (-16x^3)$

8) $-9x^2y + 9x^2y$

4) $-5x^3 - 4x^3$

9) $10x^2y^3 - 10x^3y^2$

5) $2x - 3y$

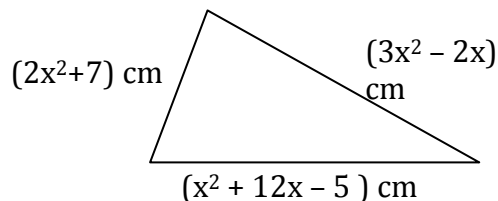
10) $5x - 3x - 8x + 6x$

B. Answer the following questions. Show your solution.

1) What is the sum of $3x^2 - 11x + 12$ and $18x^2 + 20x - 100$?

2) What is $12x^3 - 5x^2 + 3x + 4$ less than $15x^3 + 10x + 4x^2 - 10$?

3) What is the perimeter of the triangle shown at the right?



4) If you have $(100x^3 - 5x + 3)$ pesos in your wallet and you spent $(80x^3 - 2x^2 + 9)$ pesos in buying foods, how much money is left in your pocket?

5) What must be added to $3x + 10$ to get a result of $5x - 3$?

Summary

In this lesson, you learned about tiles and how to use them to represent algebraic expressions. You learned how to add and subtract terms and polynomials using these tiles. You were also able to come up with the rules in adding and subtracting polynomials. To add polynomials, simply combine similar terms. To combine similar terms, get the sum of the numerical coefficients and annex the same literal coefficients. If there is more than one term, for convenience, write similar terms in the same column. To subtract polynomials, change the sign of the subtrahend then proceed to the addition rule.

Lesson 23: Multiplying Polynomials

Time: 3 hours

Pre-requisite Concepts: Laws of exponents, Adding and Subtracting Polynomials, Distributive Property of Real Numbers

About the Lesson: In this lesson, we use the context of area to show how to multiply polynomials. Tiles will be used to illustrate the action of multiplying terms of a polynomial. Other ways of multiplying polynomials will also be taught.

Objectives:

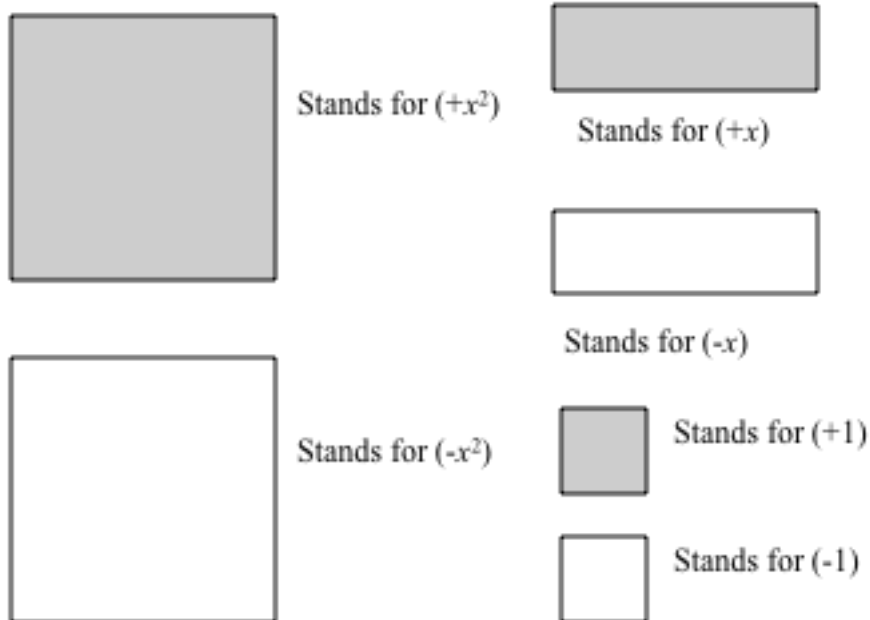
In this lesson, you should be able to:

- 1) multiply polynomials such as;
 - a) monomial by monomial,
 - b) monomial by polynomial with more than one term,
 - c) binomial by binomial,
 - d) polynomial with more than one term to polynomial with three or more terms.
- 2) solve problems involving multiplying polynomials.

Lesson Proper

I. Activity

Familiarize yourself with the following tiles:



Now, find the following products and use the tiles whenever applicable:

1) $(3x)(x)$

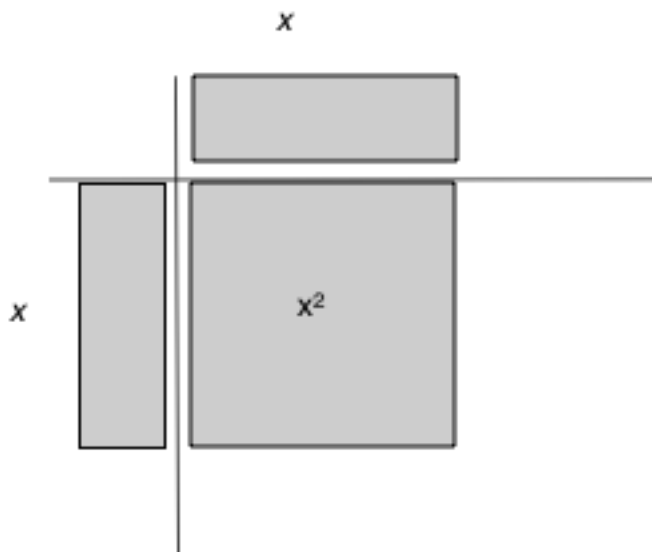
2) $(-x)(1+x)$

3) $(3-x)(x+2)$

Can you tell what the algorithms are in multiplying polynomials?

II. Questions/Points to Ponder (Post-Activity Discussion)

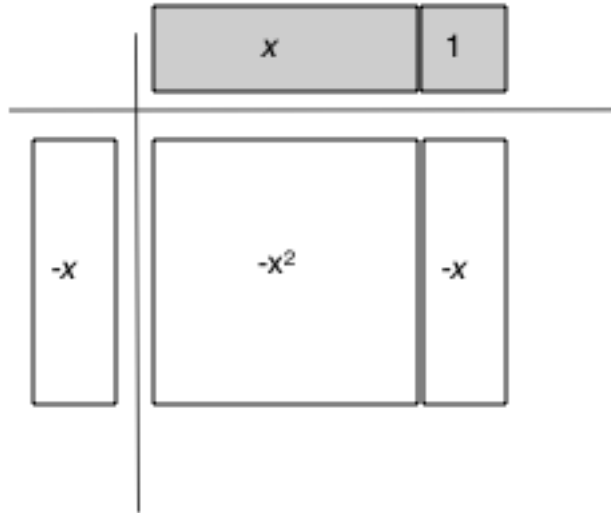
Recall the Laws of Exponents. The answer to item (1) should not be a surprise. By the Laws of Exponents, $(3x)(x) = 3x^2$. Can you use the tiles to show this product?



So, $3x^2$ is represented by three of the big shaded squares.

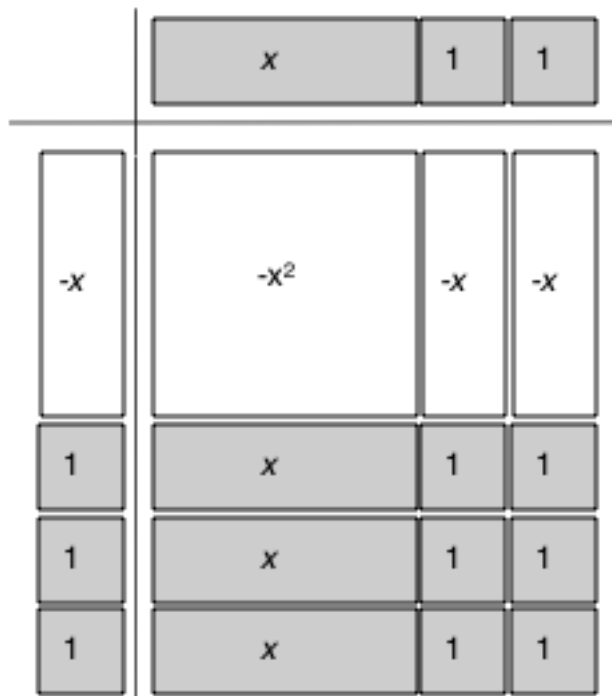


What about item (2)? The product $(-x)(1+x)$ can be represented by the following.



The picture shows that the product is $(-x^2) + (-x)$. Can you explain what happened? Recall the sign rules for multiplying.

The third item is $(3 - x)(x + 2)$. How can you use the Tiles to show the product?



$$(-x^2) + (-2x) + 3x + 6 = (-x^2) + x + 6$$

Rules in Multiplying Polynomials

A. To multiply a monomial by another monomial, simply multiply the numerical coefficients then multiply the literal coefficients by applying the basic laws of exponent.

Examples:

- 1) $(x^3)(x^5) = x^8$
- 2) $(3x^2)(-5x^{10}) = -15x^{12}$
- 3) $(-8x^2y^3)(-9xy^8) = 72x^3y^{11}$

B. To multiply monomial by a polynomial, simply apply the distributive property and follow the rule in multiplying monomial by a monomial.

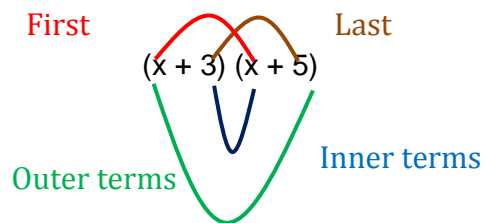
Examples:

- 1) $3x(x^2 - 5x + 7) = 3x^3 - 15x^2 + 21x$
- 2) $-5x^2y^3(2x^2y - 3x + 4y^5) = -10x^4y^4 + 15x^3y^3 - 20x^2y^8$

C. To multiply binomial by another binomial, simply distribute the first term of the first binomial to each term of the other binomial then distribute the second term to each term of the other binomial and simplify the results by combining similar terms. This procedure is also known as the F-O-I-L method or Smile method. Another way is the vertical way of multiplying which is the conventional one.

Examples

$$1) (x + 3)(x + 5) = x^2 + 8x + 15$$



$$F \rightarrow (x)(x) = x^2$$

$$O \rightarrow (x)(5) = 5x$$

$$I \rightarrow (3)(x) = 3x$$

$$L \rightarrow (3)(5) = 15$$

Since $5x$ and $3x$ are similar terms we can combine them. $5x + 3x = 8x$. The final answer is

$$x^2 + 8x + 15$$

$$2) (x - 5)(x + 5) = x^2 + 5x - 5x - 25 = x^2 - 25$$

$$3) (x + 6)^2 = (x + 6)(x + 6) = x^2 + 6x + 6x + 36 = x^2 + 12x + 36$$

$$4) (2x + 3y)(3x - 2y) = 6x^2 - 4xy + 9xy - 6y^2 = 6x^2 + 5xy - 6y^2$$

$$5) (3a - 5b)(4a + 7) = 12a^2 + 21a - 20ab - 35b$$

There are no similar terms so it is already in simplest form.

Guide questions to check whether the students understand the process or not

If you multiply $(2x + 3)$ and $(x - 7)$ by F-O-I-L method,

- a) the product of the first terms is _____.
- b) the product of the outer terms is _____.
- c) the product of the inner terms is _____.
- d) the product of the last terms is _____.
- e) Do you see any similar terms? What are they?
- f) What is the result when you combine those similar terms?
- g) The final answer is _____.

Another Way of Multiplying Polynomials

1) Consider this example.

$$\begin{array}{r} 78 \\ \times 59 \\ \hline 702 \\ 390 \\ \hline 4602 \end{array}$$

This procedure also applies the distributive property.

2) Now, consider this.

$$\begin{array}{r} 2x + 3 \\ \quad \underline{x - 7} \\ 14x + 21 \\ \underline{2x^2 + 3x} \\ 2x^2 + 17x + 21 \end{array}$$

This one looks the same as the first one.

Consider the example below.

$$\begin{array}{r} 3a - 5b \\ \underline{4a + 7} \\ 21a - 35b \\ \hline 12a^2 - 20ab \\ \underline{12a^2 - 20ab + 21a - 35b} \end{array}$$

In this case, although $21a$ and $-20ab$ are aligned, you cannot combine them because they are not similar.

D. To multiply a polynomial with more than one term by a polynomial with three or more terms, simply distribute the first term of the first polynomial to each term of the other polynomial. Repeat the procedure up to the last term and simplify the results by combining similar terms.

Examples:

$$\begin{aligned} 1) (x + 3)(x^2 - 2x + 3) &= x(x^2 - 2x + 3) - 3(x^2 - 2x + 3) \\ &= x^3 - 2x^2 + 3x - 3x^2 + 6x - 9 \\ &= \mathbf{x^3 - 5x^2 + 9x - 9} \end{aligned}$$

$$\begin{aligned} 2) (x^2 + 3x - 4)(4x^3 + 5x - 1) &= x^2(4x^3 + 5x - 1) + 3x(4x^3 + 5x - 1) - 4(4x^3 + 5x - 1) \\ &= 4x^5 + 5x^3 - x^2 + 12x^4 + 15x^2 - 3x - 16x^3 - 20x + 4 \\ &= \mathbf{4x^5 + 12x^4 - 11x^3 + 14x^2 - 23x + 4} \end{aligned}$$

$$\begin{aligned} 3) (2x - 3)(3x + 2)(x^2 - 2x - 1) &= (6x^2 - 5x - 6)(x^2 - 2x - 1) \\ &= \mathbf{6x^4 - 17x^3 - 22x^2 + 17x + 6} \end{aligned}$$

**Do the distribution one by one.*

III. Exercises

A. Simplify each of the following by combining like terms.

1) $6x + 7x$

2) $3x - 8x$

3) $3x - 4x - 6x + 2x$

4) $x^2 + 3x - 8x + 3x^2$

5) $x^2 - 5x + 3x - 15$

B. Call a student or ask for volunteers to recite the basic laws of exponent but focus more on the “product of a power” or ”multiplying with the same base”. Give follow up exercises through flashcards.

1) $x^{12} \div x^5$

2) $a^{-10} \cdot a^{12}$

3) $x^2 \cdot x^3$

4) $2^2 \cdot 2^3$

5) $x^{100} \cdot x$

C. Answer the following.

1) Give the product of each of the following.

a) $(12x^2y^3z)(-13ax^3z^4)$

b) $2x^2(3x^2 - 5x - 6)$

c) $(x - 2)(x^2 - x + 5)$

2) What is the area of the square whose side measures $(2x - 5)$ cm? (*Hint: Area of the square = s^2*)

3) Find the volume of the rectangular prism whose length, width and height are $(x + 3)$ meter, $(x - 3)$ meter and $(2x + 5)$ meter. (*Hint: Volume of rectangular prism = $l \times w \times h$*)

4) If I bought $(3x + 5)$ pencils which cost $(5x - 1)$ pesos each, how much will I pay for it?

Summary

In this lesson, you learned about multiplying polynomials using different approaches: using the Tiles, using the FOIL, and using the vertical way of multiplying numbers.

Lesson 24: Dividing Polynomials

Time: 3 hours

Pre-requisite Concepts: Addition, Subtraction, and Multiplication of Polynomials

About the Lesson: In this lesson, students will continue to work with Tiles to help reinforce the association of terms of a polynomial with some concrete objects, hence helping them remember the rules for dividing polynomials.

Objectives:

In this lesson, the students must be able to:

- 1) divide polynomials such as:
 - a) polynomial by a monomial and
 - b) polynomial by a polynomial with more than one term.
- 2) solve problems involving division of polynomials.

Lesson Proper

I. Activity 1:

Decoding

***“ I am the father of Archimedes.” Do you know my name?
Find it out by decoding the hidden message below.***

Match Column A with its answer in Column B to know the name of Archimedes' father. Put the letter of the correct answer in the space provided below.

Column A (Perform the indicated operation)

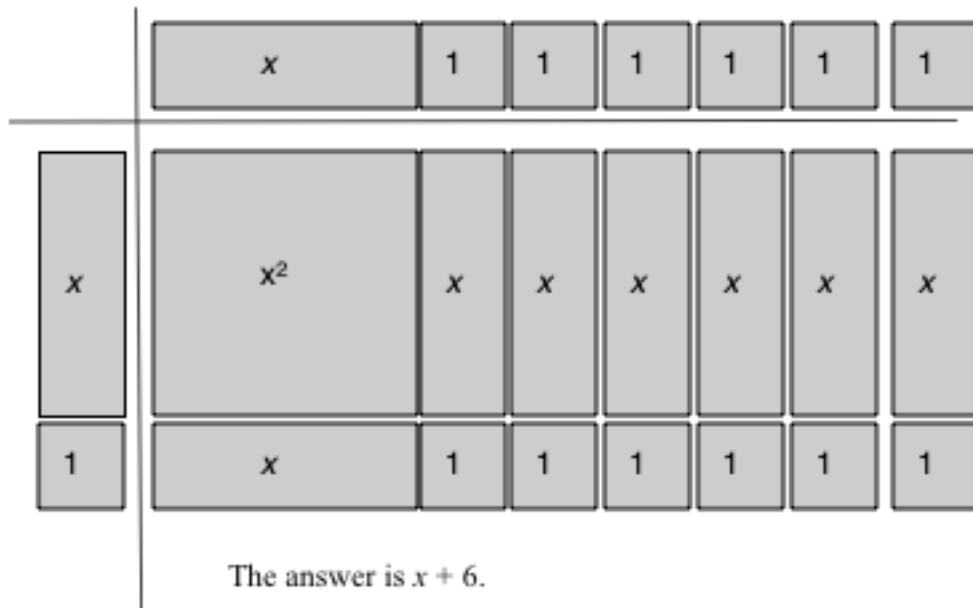
Column B

1)	$(3x^2 - 6x - 12) + (x^2 + x + 3)$	S	$4x^2 + 12x + 9$
2)	$(2x - 3)(2x + 3)$	H	$4x^2 - 9$
3)	$(3x^2 + 2x - 5) - (2x^2 - x + 5)$	I	$x^2 + 3x - 10$
4)	$(3x^2 + 4) + (2x - 9)$	P	$4x^2 - 5x - 9$
5)	$(x + 5)(x - 2)$	A	$2x^2 - 3x + 6$
6)	$3x^2 - 5x + 2x - x^2 + 6$	E	$4x^2 - 6x - 9$
7)	$(2x + 3)(2x + 3)$	D	$3x^2 + 2x - 5$
		V	$5x^3 - 5$

1 2 3 4 5 6 7

Activity 2.

Recall the Tiles. We can use these tiles to divide polynomials of a certain type. Recall also that division is the reverse operation of multiplication. Let's see if you can work out this problem using Tiles: $(x^2 + 7x + 6) \div (x + 1)$



II. Questions/Points to Ponder (Post-Activity Discussion)

The answer to Activity 1 is PHIDIAS. Di you get it? If not, what went wrong?

In Activity 2, note that the dividend is under the horizontal bar similar to the long division process on whole numbers.

Rules in Dividing Polynomials

To divide polynomial by a monomial, simply divide each term of the polynomial by the given divisor.

Examples:

1) Divide $12x^4 - 16x^3 + 8x^2$ by $4x^2$

$$\begin{aligned} \text{a) } & \frac{12x^4 - 16x^3 + 8x^2}{4x^2} \\ &= \frac{12x^4}{4x^2} - \frac{16x^3}{4x^2} + \frac{8x^2}{4x^2} \\ &= 3x^2 - 4x + 2 \end{aligned}$$

$$\begin{array}{r} 3x^2 - 4x + 2 \\ 4x^2 \overline{) 12x^4 - 16x^3 + 8x^2} \\ \underline{12x^4} \\ -16x^3 \\ \underline{-16x^3} \\ 8x^2 \\ 8x^2 \\ \hline 0 \end{array}$$

$$2) \text{ Divide } 15x^4y^3 + 25x^3y^3 - 20x^2y^4 \text{ by } -5x^2y^3 = \frac{15x^4y^3}{-5x^2y^3} + \frac{25x^3y^3}{-5x^2y^3} - \frac{20x^2y^4}{-5x^2y^3}$$

$$= -3x^2 - 5x + 4y$$

To divide polynomial by a polynomial with more than one term (by long division), simply follow the procedure in dividing numbers by long division.

These are some suggested steps to follow:

- 1) Check the dividend and the divisor if it is in standard form.
- 2) Set-up the long division by writing the division symbol where the divisor is outside the division symbol and the dividend inside it.
- 3) You may now start the Division, Multiplication, Subtraction and Bring Down cycle.
- 4) You can stop the cycle when:
 - a) the quotient (answer) has reached the constant term.
 - b) the exponent of the divisor is greater than the exponent of the dividend

Examples:

1) Divide 2485 by 12.

$$\begin{array}{r} 207 \\ 12 \overline{) 2485} \\ \underline{24} \\ 85 \\ \underline{84} \\ 1 \end{array}$$

r. 1
or $207\frac{1}{12}$

2) Divide $x^2 - 3x - 10$ by $x + 2$

$$\begin{array}{r} x - 5 \\ x + 2 \overline{) x^2 - 3x - 10} \\ \underline{x^2 + 2x} \\ -5x - 10 \\ \underline{-5x - 10} \\ 0 \end{array}$$

- 1) divide x^2 by x and put the result on top
- 2) multiply that result to $x + 2$
- 3) subtract the product to the dividend
- 4) bring down the remaining term/s
- 5) repeat the procedure from 1.

3) Divide $x^3 + 6x^2 + 11x + 6$ by $x - 3$

$$\begin{array}{r}
 x^2 - 3x + 2 \\
 x - 3 \overline{) x^3 - 6x^2 + 11x - 6} \\
 \underline{x^3 - 3x^2} \\
 - 3x + 11x \\
 \underline{- 3x + 9x} \\
 2x - 6 \\
 \underline{2x - 6} \\
 0
 \end{array}$$

4) Divide $2x^3 - 3x^2 - 10x - 4$ by $2x - 1$

$$\begin{array}{r}
 x^2 - 2x - 4 - \frac{2}{2x+1} \\
 2x+1 \overline{) 2x^3 - 3x^2 - 10x - 6} \\
 \underline{2x^3 + x^2} \\
 - 4x^2 - 10x \\
 \underline{- 4x^2 - 2x} \\
 - 8x - 6 \\
 \underline{- 8x - 4} \\
 - 2
 \end{array}$$

5) Divide $x^4 - 3x^2 + 2$ by $x^2 - 2x + 3$

$$\begin{array}{r}
 x^2 + 2x - 2 + \frac{-10x+18}{x^2-2x+3} \\
 x^2 - 2x + 3 \overline{) x^4 + 0x^3 - 3x^2 + 0x + 12} \\
 \underline{x^4 - 2x^3 + 3x^2} \\
 2x^3 - 6x^2 + 0x \\
 \underline{2x^3 - 4x^2 + 6x} \\
 -2x^2 - 6x + 12 \\
 \underline{-2x^2 + 4x - 6} \\
 -10x + 18
 \end{array}$$

III. Exercises

Answer the following.

1) Give the quotient of each of the following.

a) $30x^3y^5$ divided by $-5x^2y^5$

b)
$$\frac{13x^3 - 26x^5 - 39x^7}{13x^3}$$

c) Divide $7x + x^3 - 6$ by $x - 2$

2) If I spent $(x^3 + 5x^2 - 2x - 24)$ pesos for $(x^2 + x - 6)$ pencils, how much does each pencil cost?

3) If 5 is the number needed to be multiplied by 9 to get 45, what polynomial is needed to be multiplied to $x + 3$ to get $2x^2 + 3x - 9$?

4) The length of the rectangle is x cm and its area is $(x^3 - x)$ cm². What is the measure of its width?

Summary:

In this lesson, you have learned about dividing polynomials first using the Tiles then using the long way of dividing.

Lesson 25: Special Products

Time: 3.5 hours

Prerequisite Concepts: Multiplication and Division of Polynomials

About the Lesson: This is a very important lesson. The applications come much later but the skills will always be useful from here on.

Objectives:

In this lesson, you are expected to:

find (a) inductively, using models and (b) algebraically the

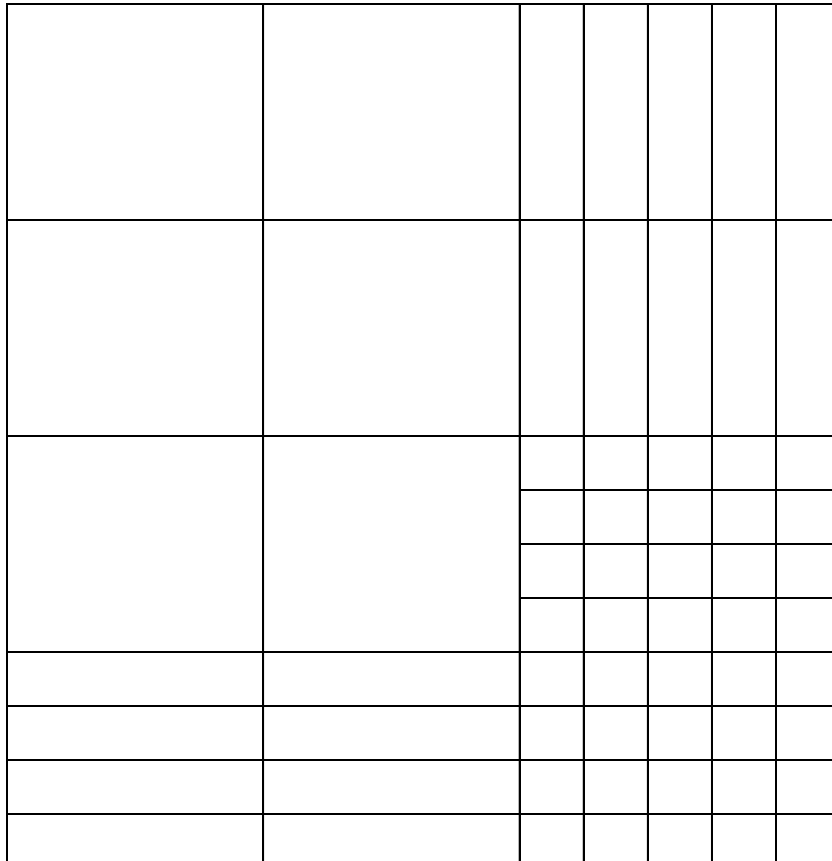
1. product of two binomials
2. product of a sum and difference of two terms
3. square of a binomial
4. cube of a binomial
5. product of a binomial and a trinomial

Lesson Proper:

A. *Product of two binomials*

I. Activity

Prepare three sets of algebra tiles by cutting them out from a page of newspaper or art paper. If you are using newspaper, color the tiles from the first set black, the second set red and the third set yellow.



Problem:

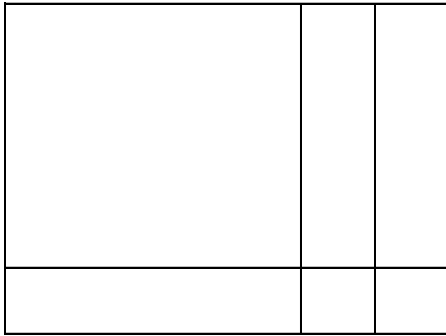
1. What is the area of a square whose sides are 2cm?
2. What is the area of a rectangle with a length of 3cm and a width of 2cm?
3. Demonstrate the area of the figures using algebra tiles.

Problem:

1. What are the areas of the different kinds of algebra tiles?
2. Form a rectangle with a length of $x + 2$ and a width of $x + 1$ using the algebra tiles. What is the area of the rectangle?

Solution:

1. x^2 , x and 1 square units.



2.

The area is the sum of all the areas of the algebra tiles.

$$\text{Area} = x^2 + x + x + x + 1 + 1 = x^2 + 3x + 2$$

Problem:

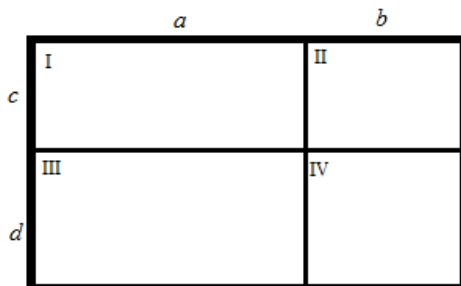
1. Use algebra tiles to find the product of the following:
 - a. $(x+2)(x+3)$
 - b. $(2x+1)(x+4)$
 - c. $(2x+1)(2x+3)$
2. How can you represent the difference $x - 1$ using algebra tiles?

Problem:

1. Use algebra tiles to find the product of the following:
 - a. $(x-1)(x-2)$
 - b. $(2x-1)(x-1)$
 - c. $(x-2)(x+3)$
 - d. $(2x-1)(x+4)$

II. Questions to Ponder

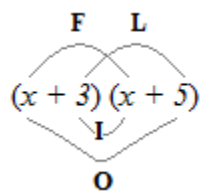
1. Using the concept learned in algebra tiles what is the area of the rectangle shown below?



2. Derive a general formula for the product of two binomials $(a+b)(c+d)$.

The area of the rectangle is equivalent to the product of $(a+b)(c+d)$ which is $ac+ad+bc+ad$. This is the general formula for the product of two binomials $(a+b)(c+d)$. This general form is sometimes called the **FOIL** method where the letters of **FOIL** stand for first, outside, inside, and last.

Example: Find the product of $(x + 3)(x + 5)$



First: $x \cdot x = x^2$
 Outside: $x \cdot 5 = 5x$
 Inside: $3 \cdot x = 3x$
 Last: $3 \cdot 5 = 15$

$$(x + 3)(x + 5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15$$

III. Exercises

Find the product using the **FOIL** method. Write your answers on the spaces provided:

1. $(x + 2)(x + 7)$
2. $(x + 4)(x + 8)$
3. $(x - 2)(x - 4)$
4. $(x - 5)(x + 1)$
5. $(2x + 3)(x + 5)$
6. $(3x - 2)(4x + 1)$
7. $(x^2 + 4)(2x - 1)$
8. $(5x^3 + 2x)(x^2 - 5)$
9. $(4x + 3y)(2x + y)$
10. $(7x - 8y)(3x + 5y)$

B. Product of a sum and difference of two terms

I. Activity

1. Use algebra tiles to find the product of the following:
 - a. $(x + 1)(x - 1)$
 - b. $(x + 3)(x - 3)$
 - c. $(2x - 1)(2x + 1)$
 - d. $(2x - 3)(2x + 3)$
2. Use the **FOIL** method to find the products of the above numbers.

II. Questions to Ponder

1. What are the products?
2. What is the common characteristic of the factors in the activity?
3. Is there a pattern for the products for these kinds of factors? Give the rule.

Concepts to Remember

The factors in the activity are called the sum and difference of two terms. Each binomial factor is made up of two terms. One factor is the **sum** of the terms and the other factor being their **difference**. The general form is $(a + b)(a - b)$.

The product of the sum and difference of two terms is given by the general formula

$$(a + b)(a - b) = a^2 - b^2.$$

III. Exercises

Find the product of each of the following:

1. $(x - 5)(x + 5)$
2. $(x + 2)(x - 2)$
3. $(3x - 1)(3x + 1)$
4. $(2x + 3)(2x - 3)$
5. $(x + y^2)(x - y^2)$
6. $(x^2 - 10)(x^2 + 10)$
7. $(4xy + 3z^3)(4xy - 3z^3)$
8. $(3x^3 - 4)(3x^3 + 4)$
9. $[(x + y) - 1][(x + y) + 1]$
10. $(2x + y - z)(2x + y + z)$

C. Square of a binomial

I. Activity

1. Using algebra tiles, find the product of the following:
 - a. $(x + 3)(x + 3)$
 - b. $(x - 2)(x - 2)$
 - c. $(2x + 1)(2x + 1)$
 - d. $(2x - 1)(2x - 1)$

- Use the **FOIL** method to find their products.

II. Questions to Ponder

- Find another method of expressing the product of the given binomials.
- What is the general formula for the square of a binomial?
- How many terms are there? Will this be the case for all squares of binomials? Why?
- What is the difference between the square of the sum of two terms from the square of the difference of the same two terms?

Concepts to Remember

The square of a binomial $a \pm b$ is the product of a binomial when multiplied to itself. The square of a binomial has a general formula, $(a \pm b)^2 = a^2 \pm 2ab + b^2$.

III. Exercises

Find the squares of the following binomials.

- $(x + 5)^2$
- $(x - 5)^2$
- $(x + 4)^2$
- $(x - 4)^2$
- $(2x + 3)^2$
- $(3x - 2)^2$
- $(4 - 5x)^2$
- $(1 + 9x)^2$
- $(x^2 + 3y)^2$
- $(3x^3 - 4y^2)^2$

D. Cube of a binomial

I. Activity

A. The cube of the binomial $(x + 1)$ can be expressed as $(x + 1)^3$. This is equivalent to $(x + 1)(x + 1)(x + 1)$.

- Show that $(x + 1)^2 = x^2 + 2x + 1$.
- How are you going to use the above expression to find $(x + 1)^3$?
- What is the expanded form of $(x + 1)^3$?

B. Use the techniques outlined above, to find the following:

- $(x + 2)^2$
- $(x - 1)^2$
- $(x - 2)^2$

II. Questions to Ponder

- How many terms are there in each of the cubes of binomials?
- Compare your answers in numbers 1 and 2?
 - What are similar with the first term? How are they different?

- b. What are similar with the second terms? How are they different?
 - c. What are similar with the third terms? How are they different?
 - d. What are similar with the fourth terms? How are they different?
3. Craft a rule for finding the cube of the binomial in the form $(x + a)^3$. Use this rule to find $(x + 3)^3$. Check by using the method outlined in the activity.
4. Compare numbers 1 and 3 and numbers 2 and 4.
 - a. What are the similarities for each of these pairs?
 - b. What are their differences?
5. Craft a rule for finding the cube of a binomial in the form $(x - a)^3$. Use this rule to find $(x - 4)^3$.
6. Use the method outlined in the activity to find $(2x + 5)^3$. Can you apply the rule you made in number 3 for getting the cube of this binomial? If not, modify your rule and use it to find $(4x + 1)^3$.

Concepts to Remember

The cube of a binomial has the general form, $a \pm b$ ³ = $a^3 \pm 3a^2b + 3ab^2 \pm b^3$.

III. Exercises

Expand.

1. $x + 5$ ³
2. $x - 5$ ³
3. $x + 7$ ³
4. $x - 6$ ³
5. $2x + 1$ ³
6. $3x - 2$ ³
7. $x^2 - 1$ ³
8. $x + 3y$ ³
9. $4xy + 3$ ³
10. $2p - 3q^2$ ³

E. Product of a binomial and a trinomial

I. Activity

In the previous activity, we have tried multiplying a trinomial with a binomial. The resulting product then had four terms. But, the product of a trinomial and a binomial **does not always** give a product of four terms.

1. Find the product of $x^2 - x + 1$ and $x + 1$.
2. How many terms are in the product?

3. What trinomial should be multiplied to $x - 1$ to get $x^3 - 1$?
4. Is there a trinomial that can be multiplied to $x - 1$ to get $x^3 + 1$?
5. Using the methods outlined in the previous problems, what should be multiplied to $x + 2$ to get $x^3 + 8$? Multiplied to $x - 3$ to get $x^3 - 27$?

II. Questions to Ponder

1. What factors should be multiplied to get the product $x^3 + a^3$? $x^3 - a^3$?
2. What factors should be multiplied to get $27x^3 + 8$?

Concepts to Remember

The product of a trinomial and a binomial can be expressed as the sum or difference of two cubes if they are in the following form.

$$\begin{array}{l} a^2 - ab + b^2 \quad a + b = a^3 + b^3 \\ a^2 + ab + b^2 \quad a - b = a^3 - b^3 \end{array}$$

III. Exercises

A. Find the product.

1. $x^2 - 3x + 9 \quad x + 3$
2. $x^2 + 4x + 16 \quad x - 4$
3. $x^2 - 6x + 36 \quad x + 6$
4. $x^2 + 10x + 100 \quad x - 10$
5. $4x^2 + 10x + 25 \quad 2x - 5$
6. $9x^2 + 12x + 16 \quad 3x - 4$

B. What should be multiplied to the following to get a sum/difference of two cubes? Give the product.

1. $x - 7$
2. $x + 8$
3. $4x + 1$
4. $5x - 3$
5. $x^2 + 2x + 4$
6. $x^2 - 11x + 121$
7. $100x^2 + 30x + 9$
8. $9x^2 - 21x + 49$

Summary: You learned plenty of special products and techniques in solving problems that require special products.